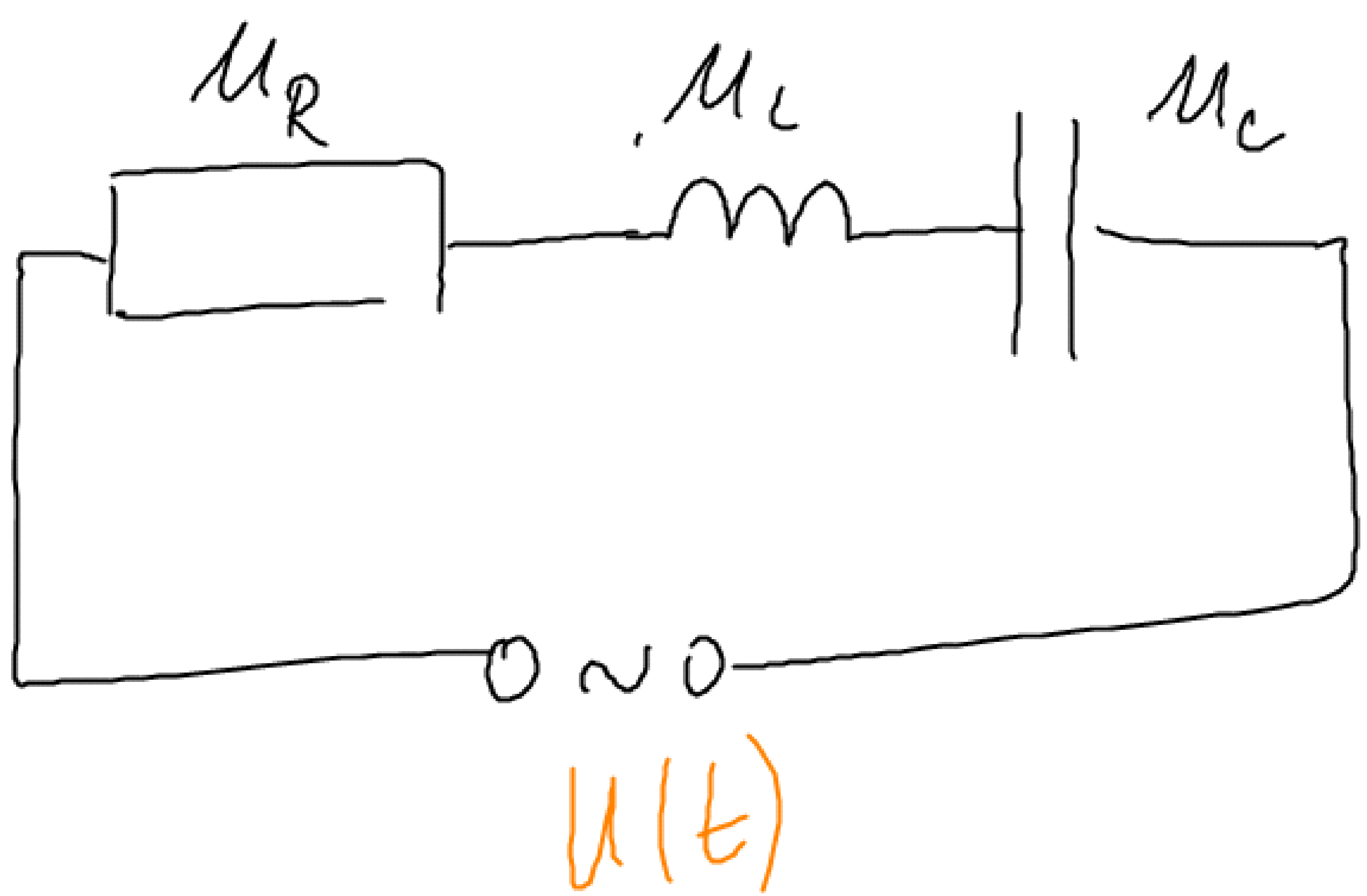
DIFERENCIÁLNÍ ROVNICE

7) sériový RLC obvod, který je buzený el. napětím $U_0 \cos \omega t$; $U_0 = \text{konst}$

$R, L, C = \text{konst}$



u_R, u_L, u_C ... okamžité hodnoty

$$u_R + u_L + u_C = U(t)$$

$$Ri + L\frac{di}{dt} + \frac{q}{C} = U(t)$$

$$i = \frac{dq}{dt}$$

$$R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C} = U(t)$$

I. řešení HOMOGENNÍ ROVNICI (tj. pravá strana nulová)

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

předpoklad řešení ve tvaru: $q = A e^{\lambda t} \quad A \in \mathbb{R} \setminus \{0\}$

$$\frac{dq}{dt} = A \cdot \lambda \cdot e^{\lambda t}$$

$$\frac{d^2 q}{dt} = A \cdot \lambda^2 e^{\lambda t}$$

$$L \cdot A \lambda^2 e^{\lambda t} + R \cdot A \cdot \lambda \cdot e^{\lambda t} + \frac{1}{C} \cdot A \cdot e^{\lambda t} = 0 \Big/ : \left(A \cdot e^{\lambda t}\right)$$

$$L \lambda^2 + R \lambda + \frac{1}{C} = 0$$

$$\lambda_{1,2} = \frac{-R \pm \sqrt{R^2 - 4\frac{L}{C}}}{2L} = -\frac{R}{2L} \pm \frac{1}{2L}\sqrt{R^2 - 4\frac{L}{C}} =$$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

cíl: el. obvodem teče el. proud $\Leftrightarrow$ „el. obvod kmitá" $\Leftrightarrow$ $\underbrace{\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}}$ $\Rightarrow \lambda_{1,2} \in \mathbb{C} \Rightarrow$

$\Rightarrow A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ lze psát pomocí Eulerova vztahu jako součet goniom. fcí $\underline{\sin \omega t}$ a $\underline{\cos \omega t}$ (viz 6,)

$$\Rightarrow \lambda_{1,2} = -\frac{R}{2L} \pm j \sqrt{\underbrace{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}_{> 0}} \qquad j^2 = -1$$

$$\lambda_{1,2} = -\frac{R}{2L} \pm j\omega_1$$

$$q_h = A_1 e^{\left(-\frac{R}{2L}+j\omega_1\right)t} + A_2 e^{\left(-\frac{R}{2L}-j\omega_1\right)t} \overset{\text{viz 6}}{=}$$

$$= \underline{A}\, e^{-\frac{R}{2L}t}\left(\underline{B}\cos\omega_1 t + \underline{C}\sin\omega_1 t\right)$$

OBECNÉ ŘEŠENÍ HOMOGENNÍ RCE

$A, B, C \in \mathbb{R}$ ... z poč. podmínek

II. ŘEŠENÍ RCE S NENULOVOU PRAVOU STRANOU

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = U_0 \cos\omega t$$

předpoklad: $q_p = E \sin\omega t + D\cos\omega t$

$$\frac{dq_p}{dt} = E\omega\cos\omega t - D\omega\sin\omega t$$

$$\frac{d^2q_p}{dt^2} = -E\omega^2\sin\omega t - D\omega^2\cos\omega t$$

$$L\left(-E\omega^2 \sin\omega t - D\omega^2 \cos\omega t\right) + R\left(E\omega\cos\omega t - D\omega \sin\omega t\right) +$$

$$+ \frac{1}{C}\left(E \sin\omega t + D\cos\omega t\right) = U_0 \cos\omega t$$

$$-LE\omega^2 - RD\omega + \frac{1}{C}E = 0$$

$$-LD\omega^2 + RE\omega + \frac{1}{C}D = U_0$$

$$E\left(\frac{1}{C} - L\omega^2\right) - R\omega D = 0$$

$$ER\omega \qquad + D\left(\frac{1}{C} - L\omega^2\right) = U_0$$

$$\rightarrow D = E\,\frac{\frac{1}{C} - L\omega^2}{R\omega}$$

$$ER\omega + E\frac{\left(\frac{1}{C} - L\omega^2\right)^2}{R\omega} = U_0$$

$$E\left(\frac{R^2\omega^2 + \left(\frac{1}{C} - L\omega^2\right)^2}{R\omega}\right) = U_0$$

$$E = \frac{U_0 R\omega}{R^2\omega^2 + \left(\frac{1}{C} - L\omega^2\right)^2}$$

$$D = \frac{U_0\left(\frac{1}{C} - L\omega^2\right)}{R^2\omega^2 + \left(\frac{1}{C} - L\omega^2\right)^2}$$

$$\boxed{q_p = \frac{U_0 R\omega}{R^2\omega^2 + \left(\frac{1}{C} - L\omega^2\right)^2}\sin\omega t + \frac{U_0\left(\frac{1}{C} - L\omega^2\right)}{R^2\omega^2 + \left(\frac{1}{C} - L\omega^2\right)^2}\cos\omega t}$$

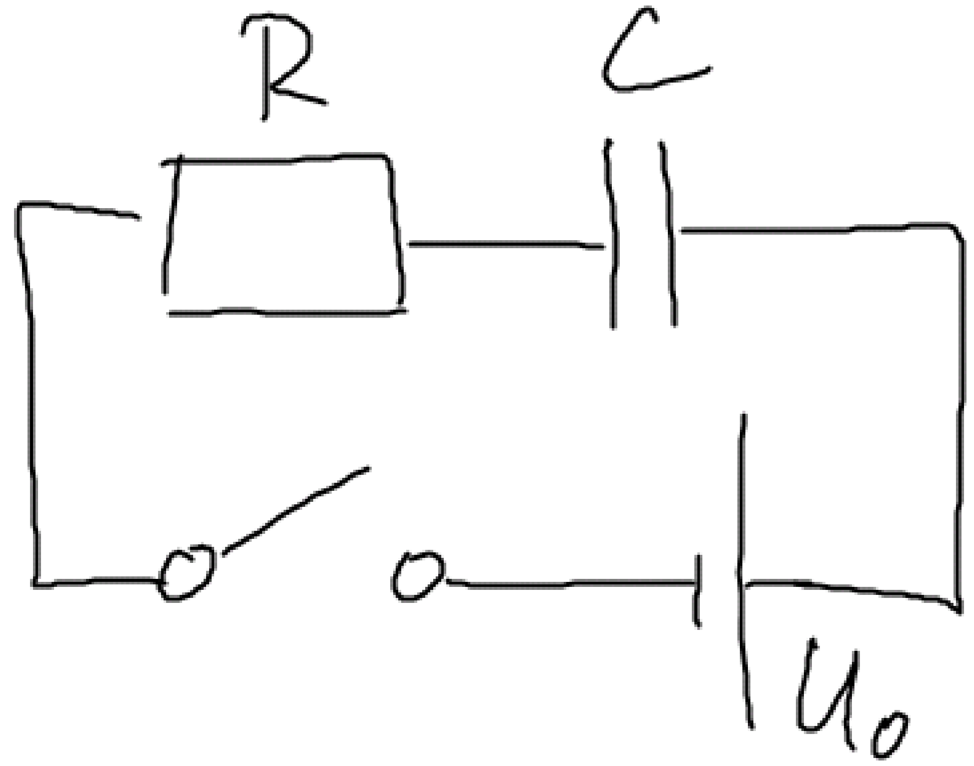
Závěr: $q(t) = q_h + q_p$

obecné řešení HOMOGENNÍ RCE

partikulární řešení NEHOMOGENNÍ RCE (nenulová pravá strana)

8, Odvoďte časový průběh el. napětí měřeného na (deskách) kondenzátorem s kapacitou $\underline{C}$ při jeho nabíjení a časový průběh el. proudu tekoucího obvodem. Kondenzátor nabíjíme přes rezistor o odporu $\underline{R}$ a je připojen ke zdroji stejnosměrného napětí $\underline{U_0}$.

$R, C, U_0 = \text{konst.}$



kondenzátor ... $u(t)$

po sepnutí spínače teče el. proud: $i = \dfrac{U_0 - u}{R}$

za dobu $\underline{\Delta t}$ projde obvodem náboj ΔQ: $\Delta Q = i\,\Delta t$

za tuto dobu se napětí na kondenzátoru zvětší o hodnotu Δu: $\Delta u = \dfrac{\Delta Q}{C}$

$$\Delta u = \frac{i\,\Delta t}{C}$$

$$\Delta u = \frac{U_0 - u}{RC}\,\Delta t$$

$$\frac{du}{dt} = \frac{u_0 - u}{RC}$$

SEPARACE PROMĚNNÝCH

$$\frac{du}{u_0 - u} = \frac{dt}{RC}$$

$$-\int \frac{-du}{u_0 - u} = \int \frac{dt}{RC}$$

$$\ln|u_0 - u| = -\frac{t}{RC} + K$$

$$u_0 - u = e^{-\frac{t}{RC} + K}$$

$$u_0 - u = K_1 \cdot e^{-\frac{t}{RC}}$$

$$u = U_0 - K_1 e^{-\frac{t}{RC}}$$

poč. podmínky: $u(0) = 0$

$$0 = U_0 - K_1 \cdot e^{0}$$

$$K_1 = U_0$$

$$\underline{\underline{u(t)}} = U_0 - U_0 e^{-\frac{t}{RC}} = \underline{\underline{U_0 \left(1 - e^{-\frac{t}{RC}}\right)}}$$

$[RC] = ?$

$[RC] = s \Longleftarrow \left[\frac{t}{RC}\right] = 1$

„zkouška": $t = 0 \Rightarrow u(0) = 0$

$t \to \infty \Rightarrow u(t) = U_0$

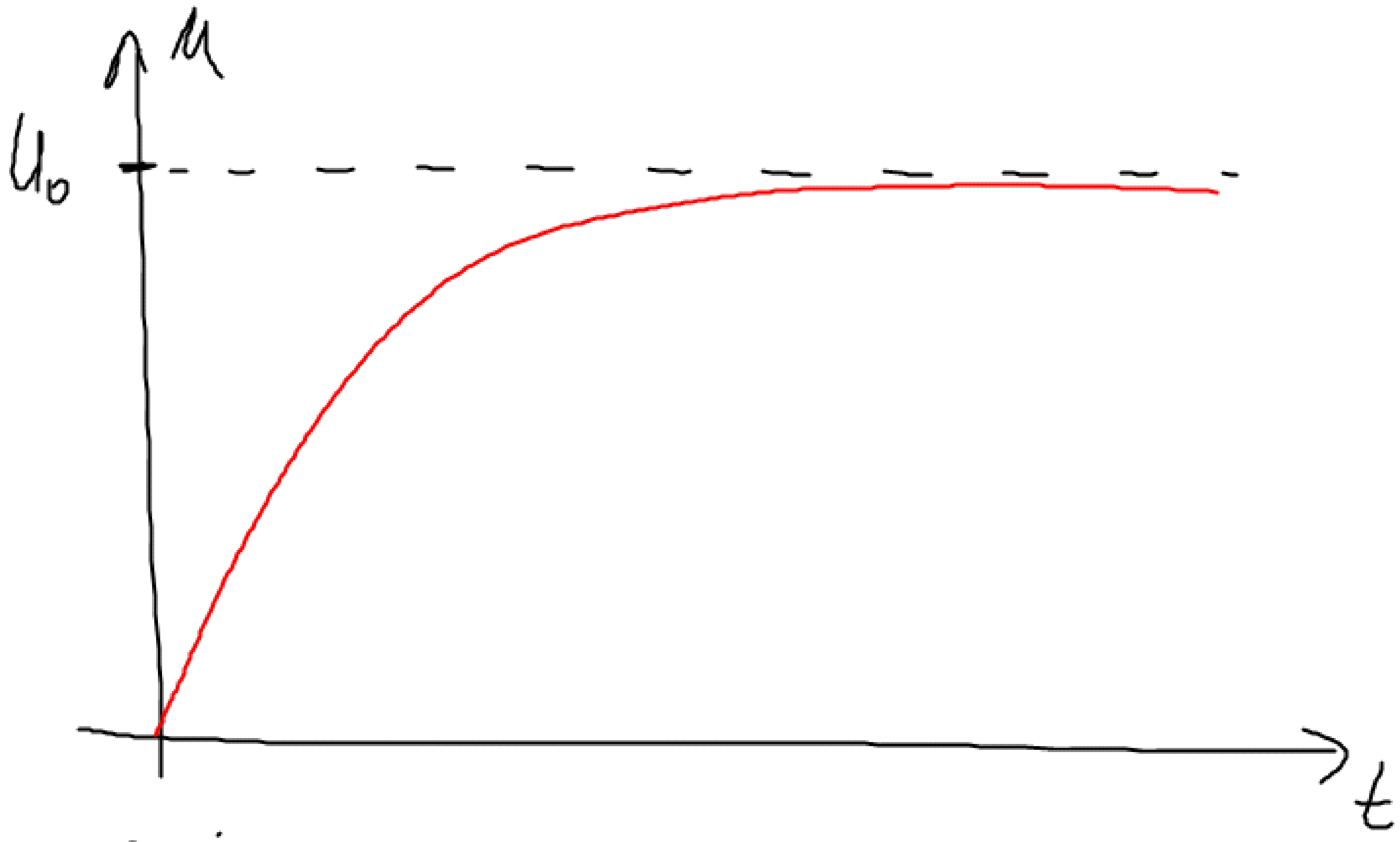
el. proud: $i = \frac{U_0 - u}{R}$

$$i = \frac{1}{R}\left(U_0 - U_0\left(1 - e^{-\frac{t}{RC}}\right)\right)$$

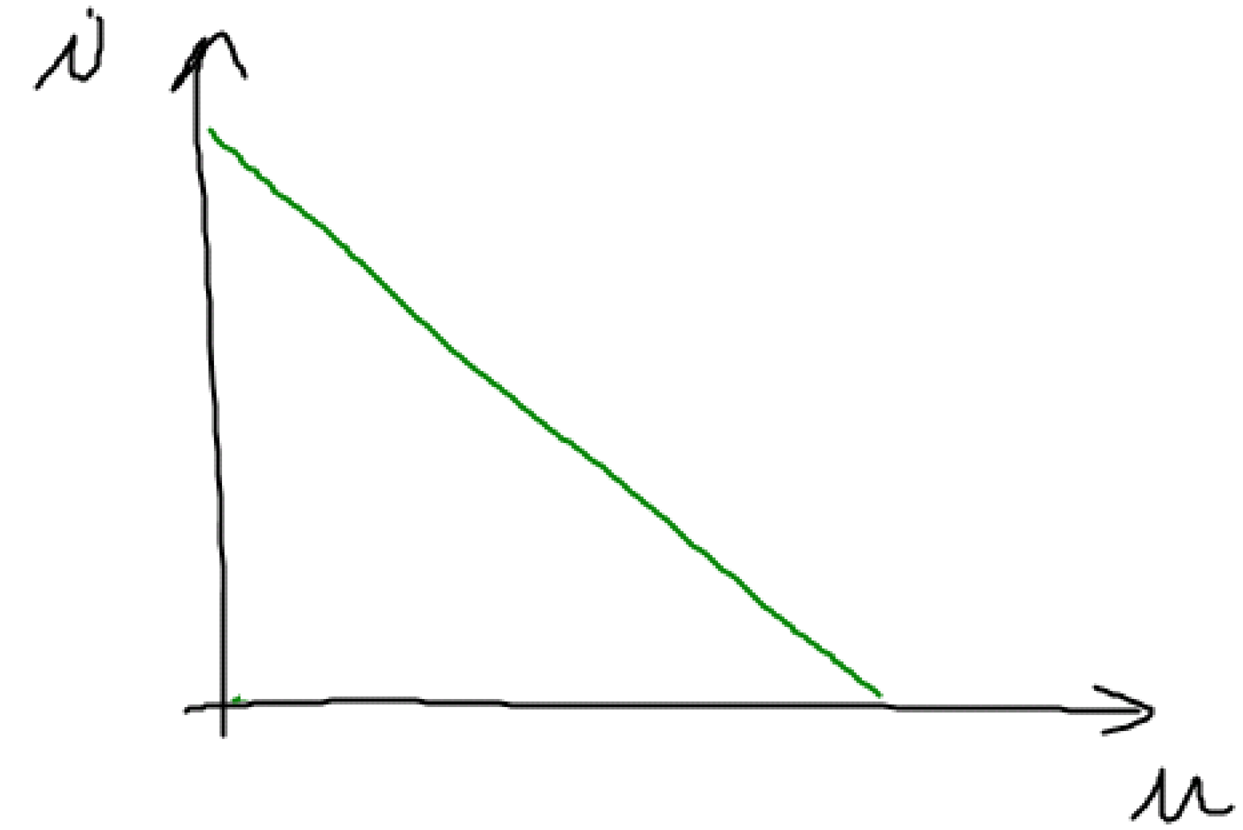
$$\underline{\underline{i = \frac{U_0}{R} e^{-\frac{t}{RC}}}}$$

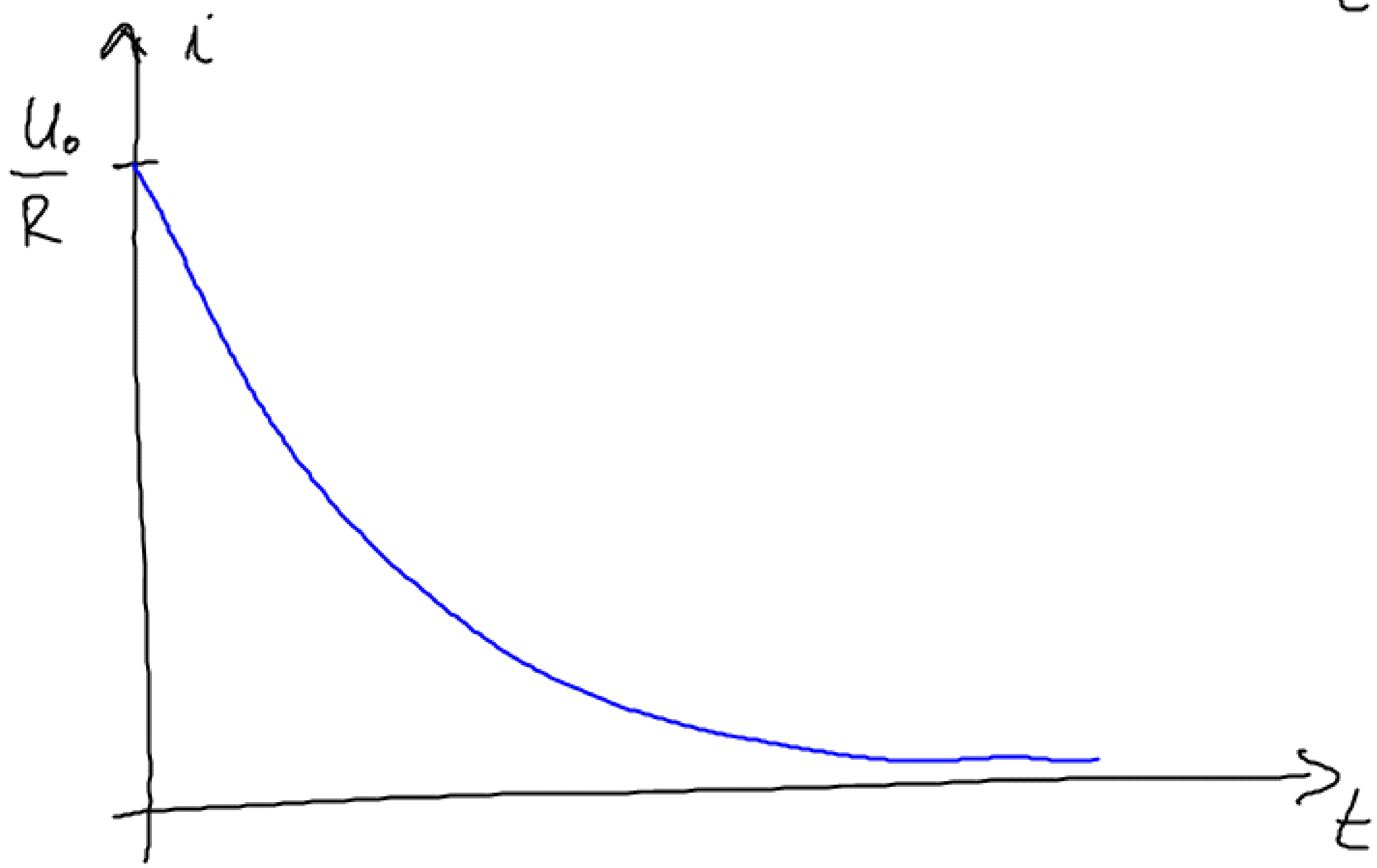
„zkouška": $t = 0 \Rightarrow i(0) = \frac{U_0}{R}$

$t \to \infty \Rightarrow i(t) = 0$



měření pomocí čidel:





$$u(t) = U_0\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i(t) = \frac{U_0}{R} \cdot e^{-\frac{t}{RC}}$$

$$e^{-\frac{t}{RC}} = \frac{iR}{U_0}$$

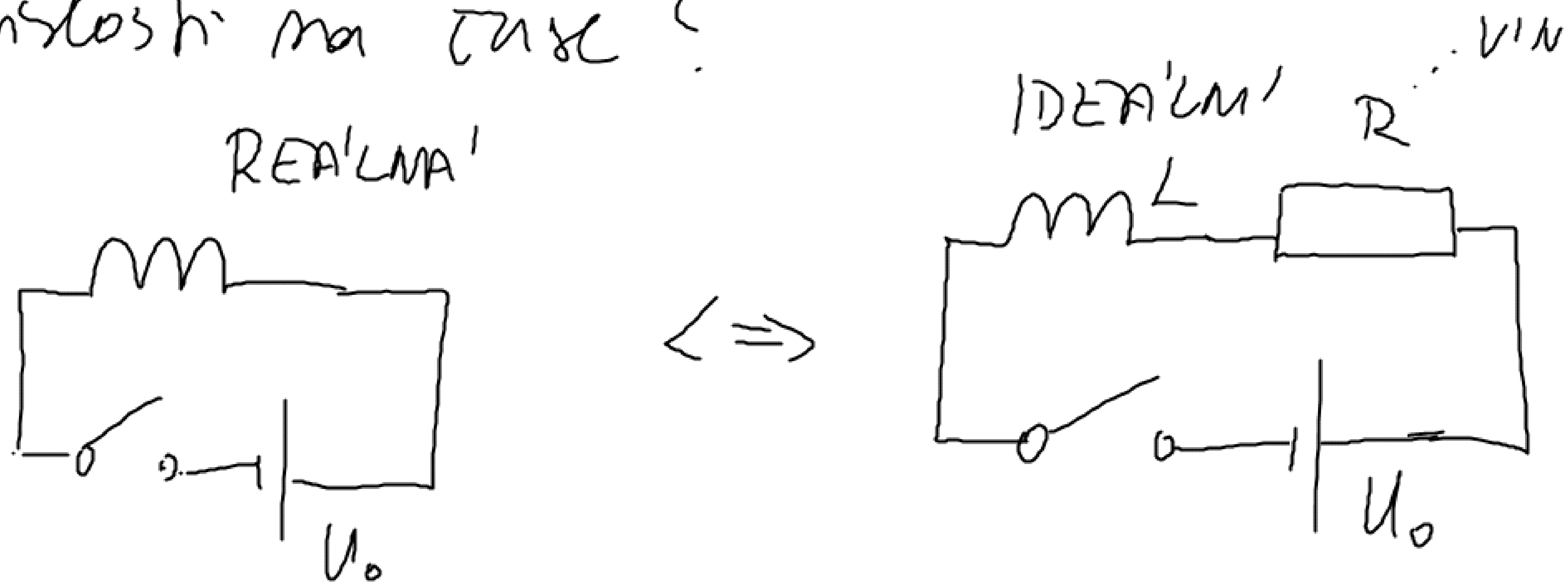
$$u = U_0\left(1 - \frac{iR}{U_0}\right)$$

$$u = U_0 - iR$$

inbo

$$i = \frac{U_0 - u}{R}$$

9) Cívku s indukčností $\underline{L}$ a ohmickým odporem $\underline{R}$ připojíme ke zdroji stejnosměrného napětí $\underline{U_0}$. Jaký bude průběh el. proudu v závislosti na čase?



$L, R = $
$= \text{konst.}$

$$U_{Li} = \frac{d\phi}{dt} = L\frac{dI}{dt}$$

$$U_L = U_0 - U_{Li} = RI$$

$$U_0 - L\frac{dI}{dt} = RI$$

(1) $$L\frac{dI}{dt} + RI = U_0 \qquad \left(L\frac{dI}{U_0 - RI} = dt\right)$$

šlo by řešit seperací proměnných, ale ukážeme metodu <u>VARIACE KONSTANT</u>, ta se používá v případě, že pravá strana ne<u>MĚNÍ KONSTANTNÍ</u>

<u>I.</u> HOMOGENNÍ RCE: $\frac{dI}{dt} + \frac{R}{L}I = 0$

předpoklad: $I = K \cdot e^{\lambda t}$

$$\frac{dI}{dt} = K \cdot \lambda e^{\lambda t}$$

$$K\lambda e^{\lambda t} + \frac{R}{L} K e^{\lambda t} = 0 \Big/ :(K \cdot e^{\lambda t}) \quad ; \quad K \neq 0 \text{ FYZIKA}$$

$$e^{\lambda t} \neq 0$$

$$\lambda + \frac{R}{L} = 0$$

$$\lambda = -\frac{R}{L}$$

řešení: $I = K \cdot e^{-\frac{R}{L}t}$

II. RCE S NENULOVOU PRAVOU STRANOU – ZA PŘEDPOKLADU, ŽE $\underline{K}$ NENÍ KONSTANTA, ALE ZÁVISÍ NA $\underline{t}$

předpoklad: $I(t) = K(t) \cdot e^{-\frac{R}{L}t} \quad (2)$

$$\frac{dI}{dt} = \frac{dK}{dt} \cdot e^{-\frac{R}{L}t} + K \cdot \left(-\frac{R}{L}\right) \cdot e^{-\frac{R}{L}t}$$

dosadit do (1):

$$L\cdot\left(\frac{dK}{dt}\cdot e^{-\frac{R}{L}t} - K\frac{R}{L}e^{-\frac{R}{L}t}\right) + R\cdot K\cdot e^{-\frac{R}{L}t} = U_0$$

$$L\frac{dK}{dt}\cdot e^{-\frac{R}{L}t} \cancel{-KR\, e^{-\frac{R}{L}t}} + \cancel{RK\cdot e^{-\frac{R}{L}t}} = U_0$$

TOTO MUSÍ NASTAT

$$\frac{dK}{dt} = \frac{U_0}{L}\cdot e^{\frac{R}{L}t}$$

$$K = \frac{U_0}{L}\cdot e^{\frac{R}{L}t}\,\frac{1}{\frac{R}{L}} + K_1$$

$$K = \frac{U_0}{R}\cdot e^{\frac{R}{L}t} + K_1$$

dosadit do (2):

$$I(t) = \left(\frac{U_0}{R} e^{\frac{R}{L}t} + K_1\right) e^{-\frac{R}{L}t}$$

$$I(t) = \frac{U_0}{R} + K_1 e^{-\frac{R}{L}t}$$ ... obecné řešení rce

poč. podmínky: $t = 0 \Rightarrow I = 0$

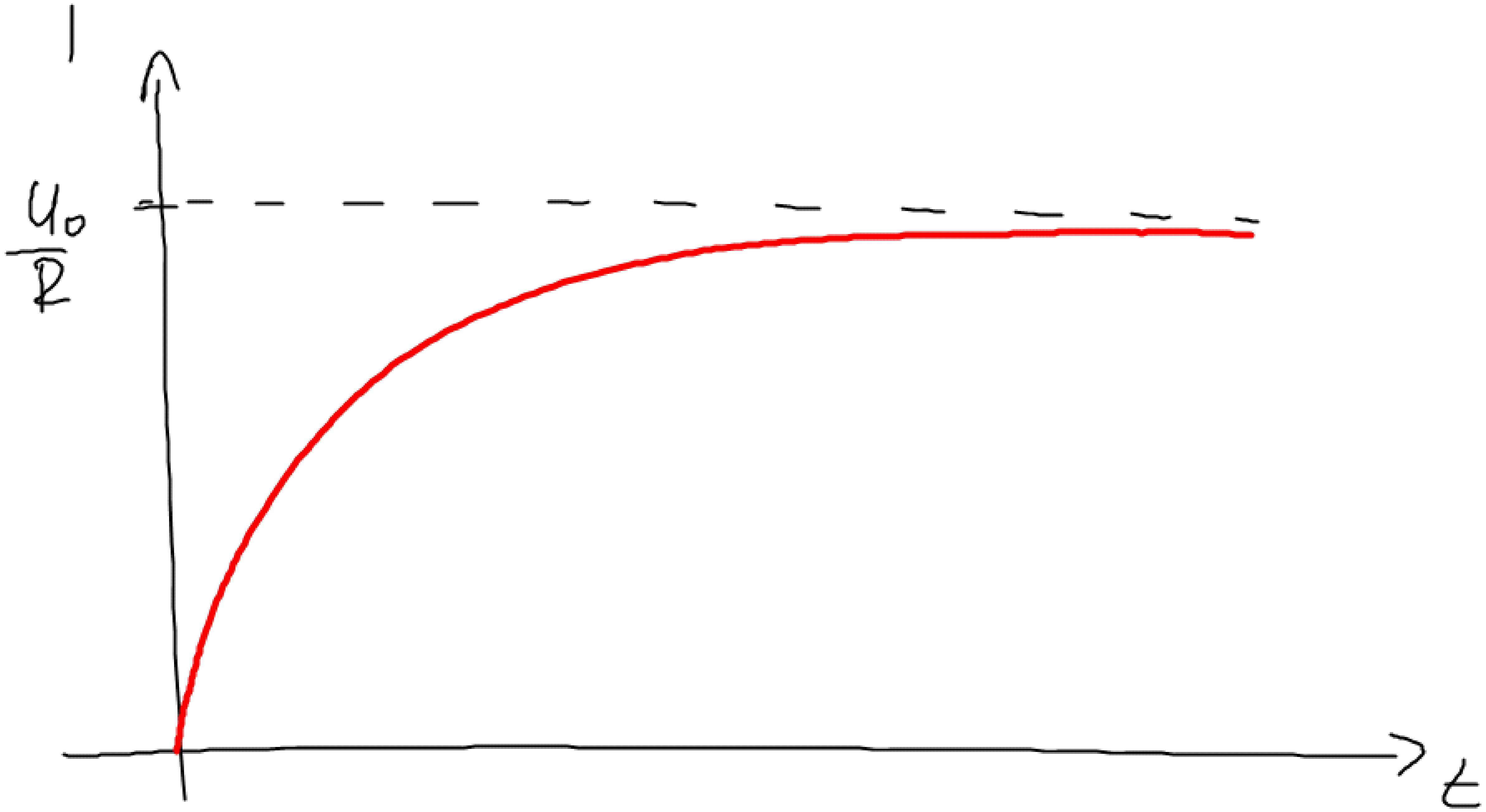
$$0 = \frac{U_0}{R} + K_1 \cdot e^0$$

$$K_1 = -\frac{U_0}{R}$$

$$\left[\frac{R}{L}\right] = s^{-1}$$

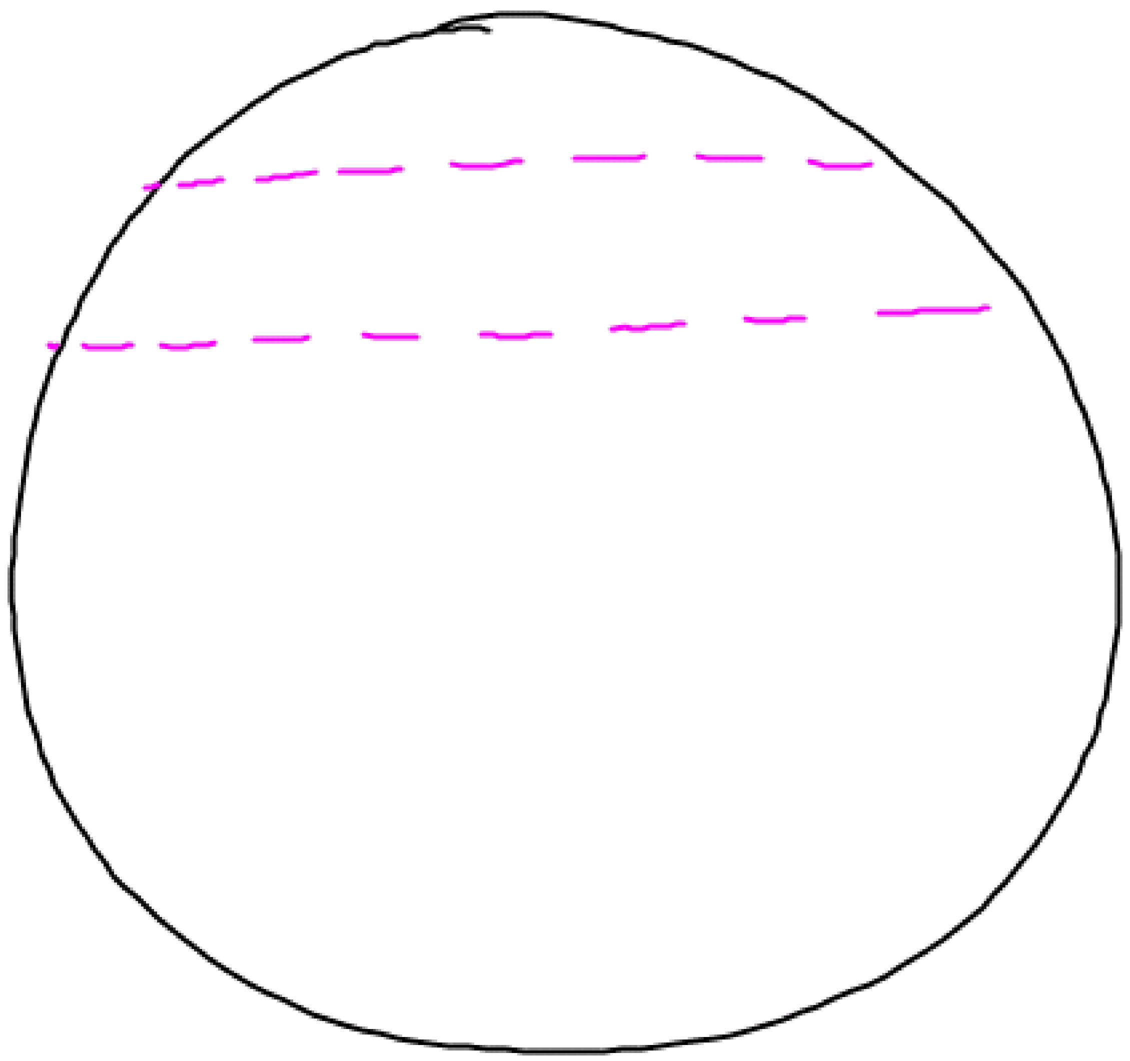
$$\left[\frac{L}{R}\right] = s$$

$$\underline{\underline{I(t) = \frac{U_0}{R}\left(1 - e^{-\frac{R}{L} \cdot t}\right)}}$$



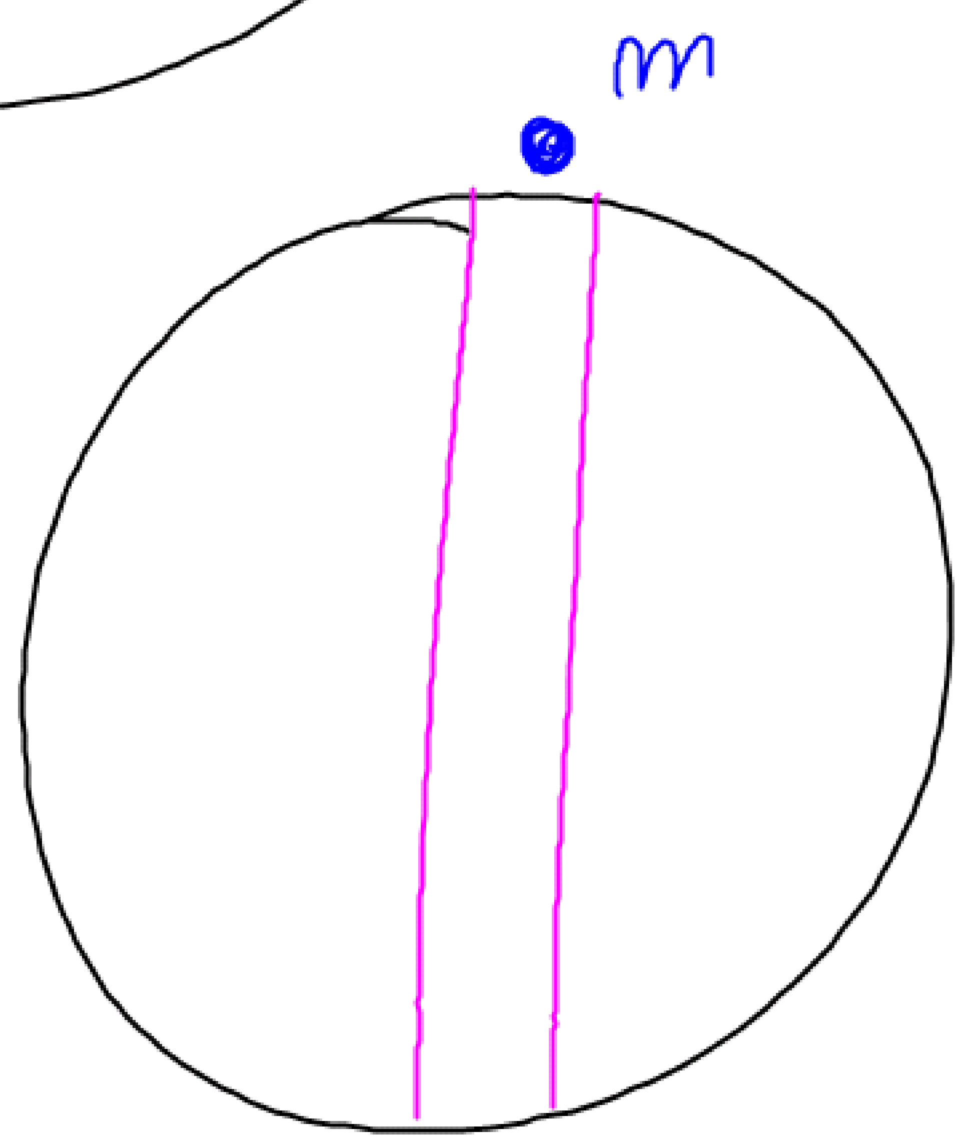
I
$\frac{U_0}{R}$
t

10) tunel v Zemi

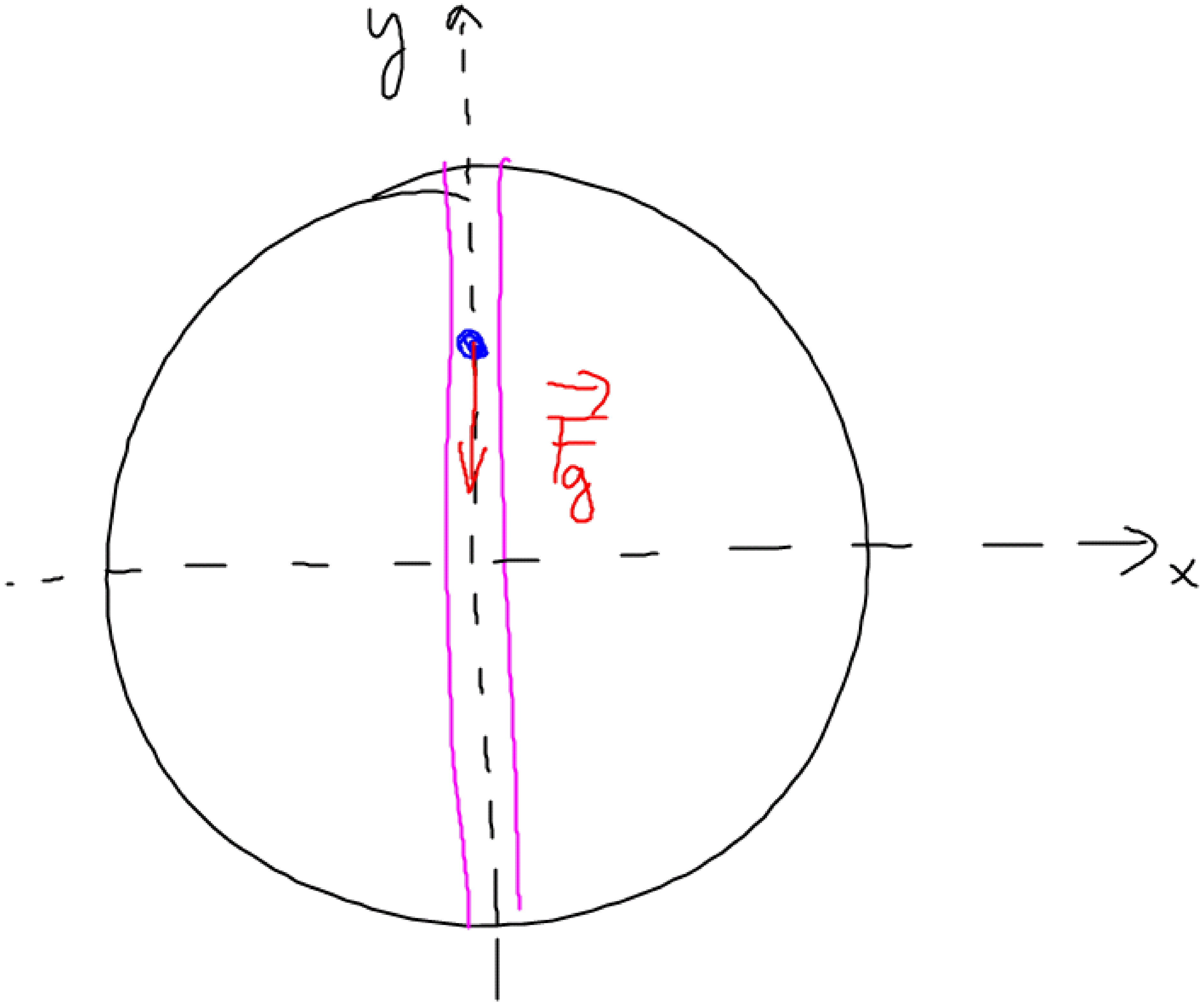


řešeno na Heuréce ZŠ

vyšší level



Za jak dlouho těleso proletí tunelem?



$$\vec{F_g} = m\vec{a}$$

$$-\beta y = m\frac{d^2y}{dt^2}$$

~ kmitání tělesa na pružině

$$m\frac{d^2y}{dt^2} + \beta y = 0$$

$$\frac{d^2y}{dt^2} + \frac{\beta}{m} y = 0$$

$$\omega^2 = \frac{\beta}{m}$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \qquad (1)$$

předpoklad: $y = Y \sin(\omega t + \varphi_0)$

$$\frac{dy}{dt} = Y\omega\cos(\omega t + \varphi_0)$$

$$\frac{d^2y}{dt^2} = -Y\omega^2 \sin(\omega t + \varphi_0)$$

(R - poloměr Země); $Y = R$

$t = 0 \Rightarrow y = R$

$R = Y \sin(0 + \varphi_0) \longrightarrow \sin\varphi_0 = 1$

$\varphi_0 = \frac{\pi}{2}$

$$y = R \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$t = 0 \Rightarrow F_g = mg = -\beta \cdot (-R)$$

$$\beta = \frac{mg}{R}$$

$$\omega^2 = \frac{\beta}{m} = \frac{g}{R}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{R}}$$

$$T = 2\pi \sqrt{\frac{R}{g}}$$

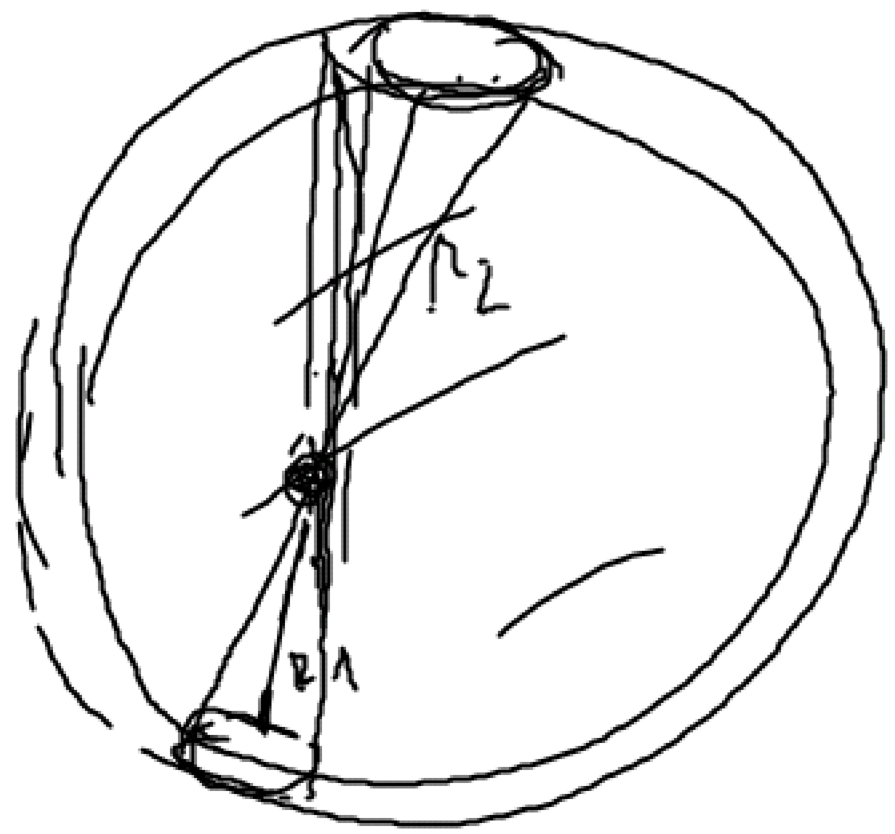
$$\underline{t_{letu}} = \frac{T}{2} = \underline{\pi \sqrt{\frac{R}{g}}}$$

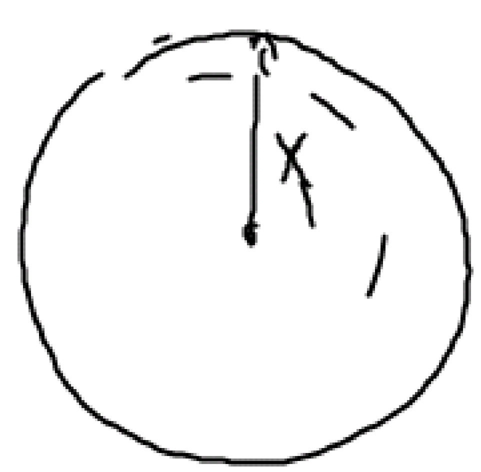
$$t = 3{,}14 \cdot \sqrt{\frac{6{,}378 \cdot 10^6}{10}}\ s$$

$$t \doteq 3{,}14 \cdot 8 \cdot 10^2\ s$$

$$t \doteq 2500\ s$$

$$\underline{t \doteq 40\ min}$$





$$a_g = \gamma \cdot \frac{M_x}{x^2} =$$

$$= \gamma \cdot \frac{\rho \cdot 4/3\,\pi x^3}{x^2}$$

$$a_g \sim x$$

11, Jaderný rozpad

závislost počtu $\underline{N}$ nerozpadlých jader v čase $\underline{t}$ na čase

$$\Delta N = -\lambda N \cdot \Delta t$$

N – počet nerozpadlých v čase $\underline{t}$

ΔN – počet rozpadlých za čase $\underline{\Delta t}$

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda dt$$

$$\ln N = -\lambda t + K$$

$$e^{-\lambda t + K} = N$$

$$N = L \cdot e^{-\lambda t}$$

poč. podmínky: $t = 0$ jДо NEROZPADLÝCH N_0

$$N_0 = L \cdot e^{-\lambda \cdot 0}$$

$$L = N_0$$

závěr: $\boxed{N = N_0 \cdot e^{-\lambda t}}$

poločas rozpadu $\underline{T}$ - doba, za kterou se STATISTICKY

rozpadne polovina částic

$$\frac{N_0}{2} = N_0 e^{-\lambda T}$$

$$\frac{1}{2} = e^{-\lambda T}$$

$$e^{\lambda T} = 2$$

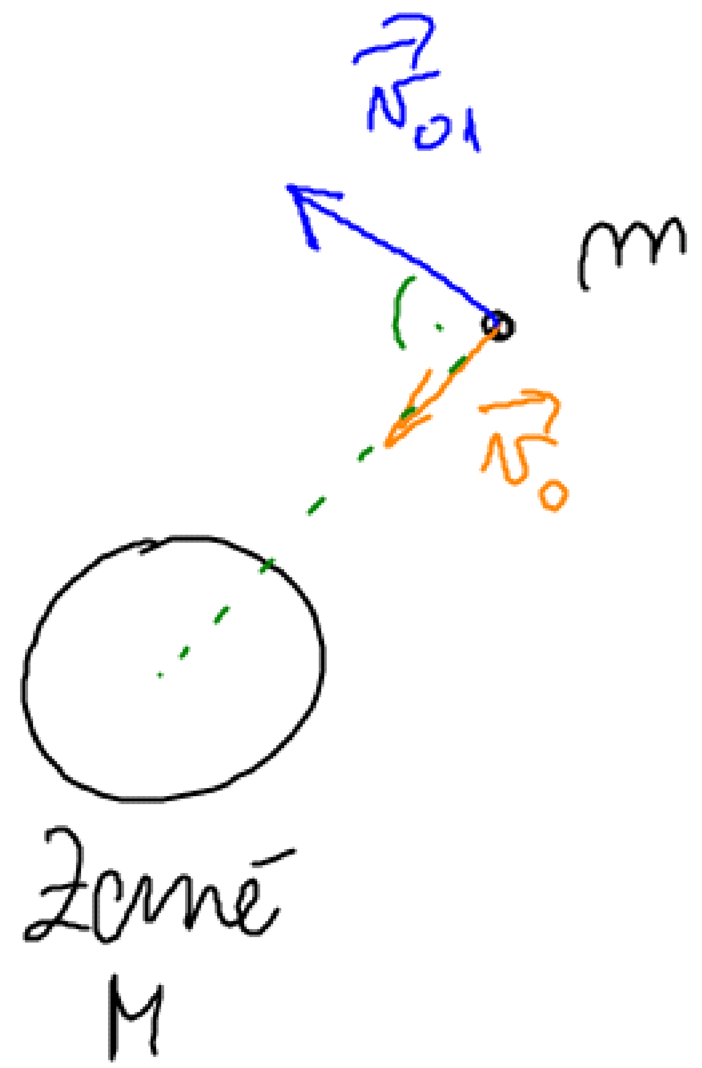
$$\ln e^{\lambda T} = \ln 2$$

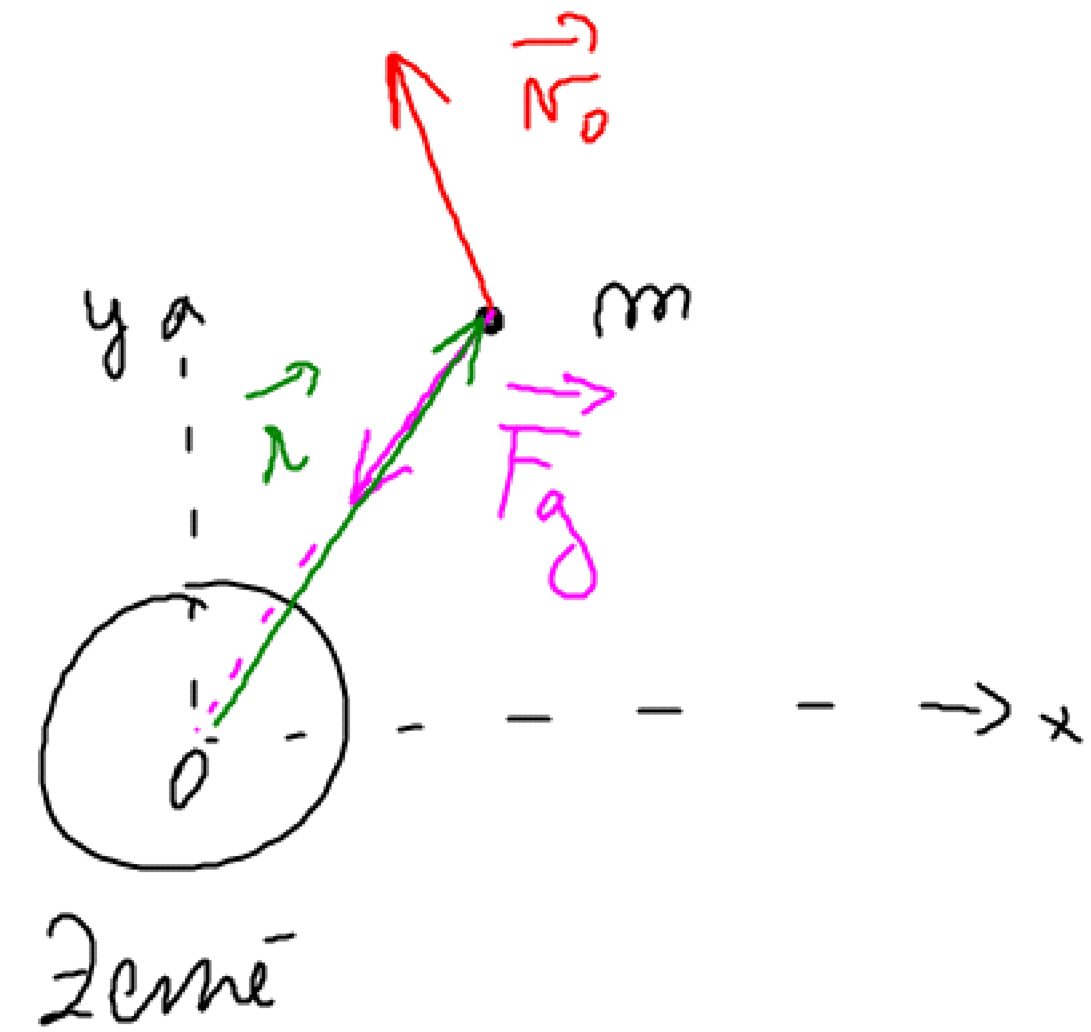
$$\lambda T \cdot \ln e = \ln 2$$

$$T = \frac{\ln 2}{\lambda}$$

12, pohyb tělesa v centrálním gr. poli Země

NEBUDE ŘEŠENO, JEN UKÁZKA





$$\vec{F}_g = \varkappa \frac{Mm}{r^2} \vec{n}_0 = -\varkappa \frac{Mm}{r^2} \frac{\vec{r}}{|\vec{r}|}$$

jednotkový vektor ve směru $\vec{F}_g$

$$\vec{F}_g = -\varkappa \frac{Mm}{r^3}\vec{r}$$

pohyb telesa : 2NZ $\Rightarrow \vec{F}_g = m\vec{a}$

$$-\varkappa \frac{M\cancel{m}}{r^3}\vec{r} = \cancel{m}\frac{d^2\vec{r}}{dt^2}$$

$\vec{r} = (x; y)$

$|\vec{r}| = \sqrt{x^2+y^2}$

$$-\varkappa \frac{M}{\left(\sqrt{x^2+y^2}\right)^3}(x; y) = \frac{d^2(x; y)}{dt^2}$$

$\Rightarrow$ 2 rce: $-\varkappa \dfrac{M}{\sqrt{(x^2+y^2)^3}}\,x = \dfrac{d^2x}{dt^2}$; $-\varkappa \dfrac{M}{\sqrt{(x^2+y^2)^3}}\,y = \dfrac{d^2y}{dt^2}$

FYZIKÁLNÍ OLYMPIÁDA

FO-30-1-D

$R = 6\,cm$

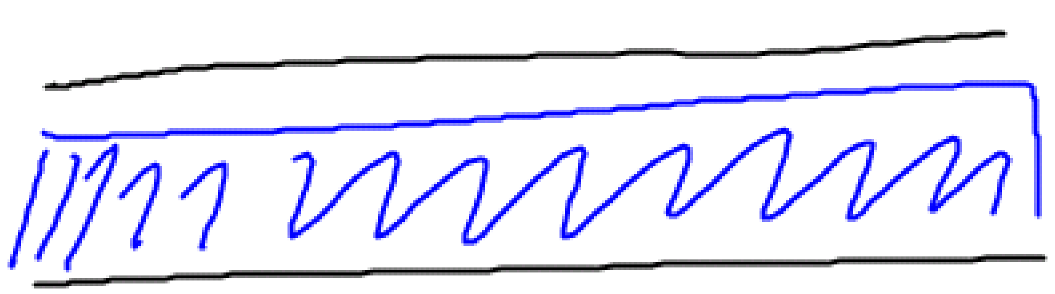
$t_1 = 12\,s$

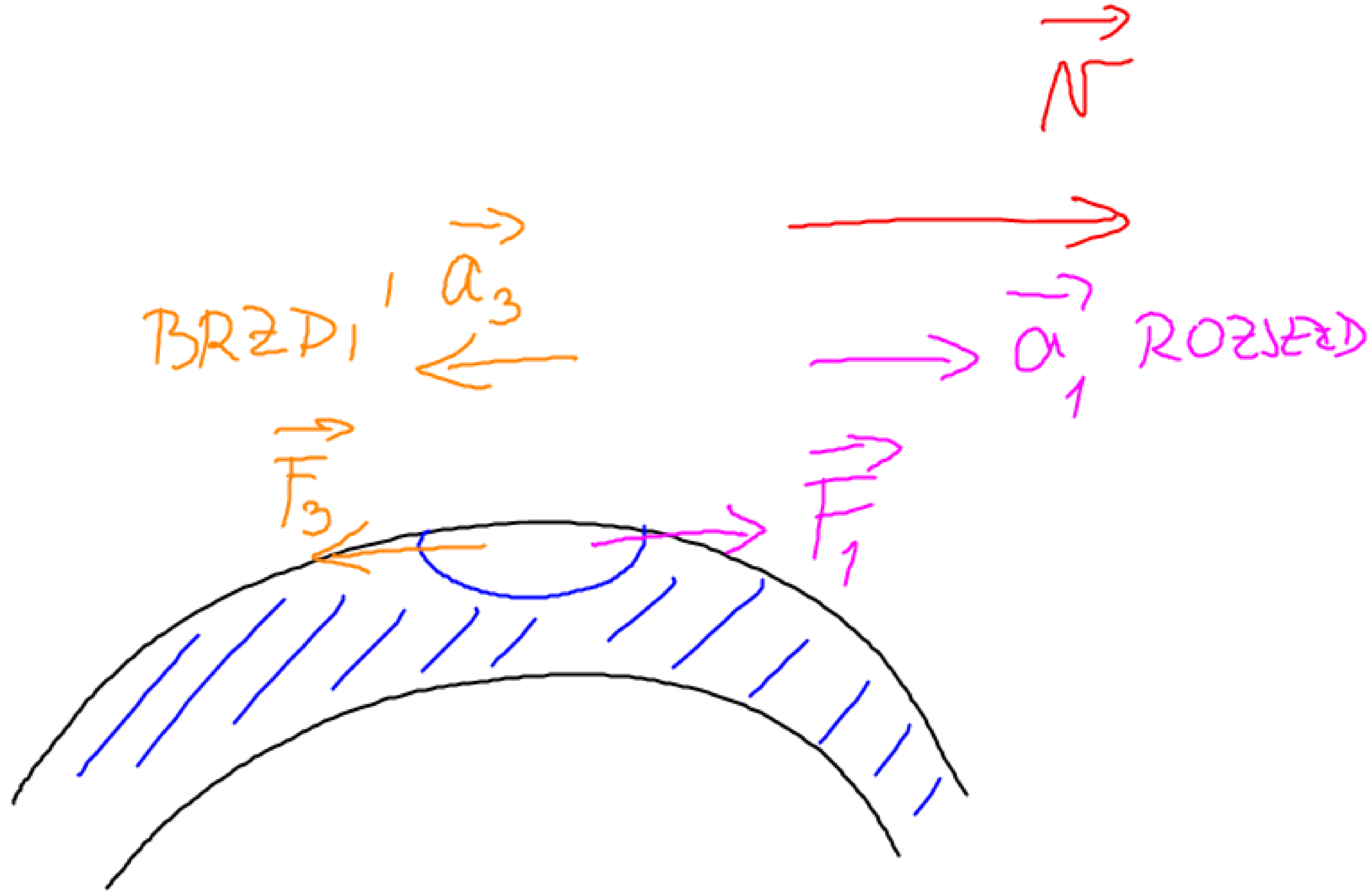
$x_1 = 12\,mm$

$t_2 = 50\,s$

$x_3 = 15\,mm$

a)

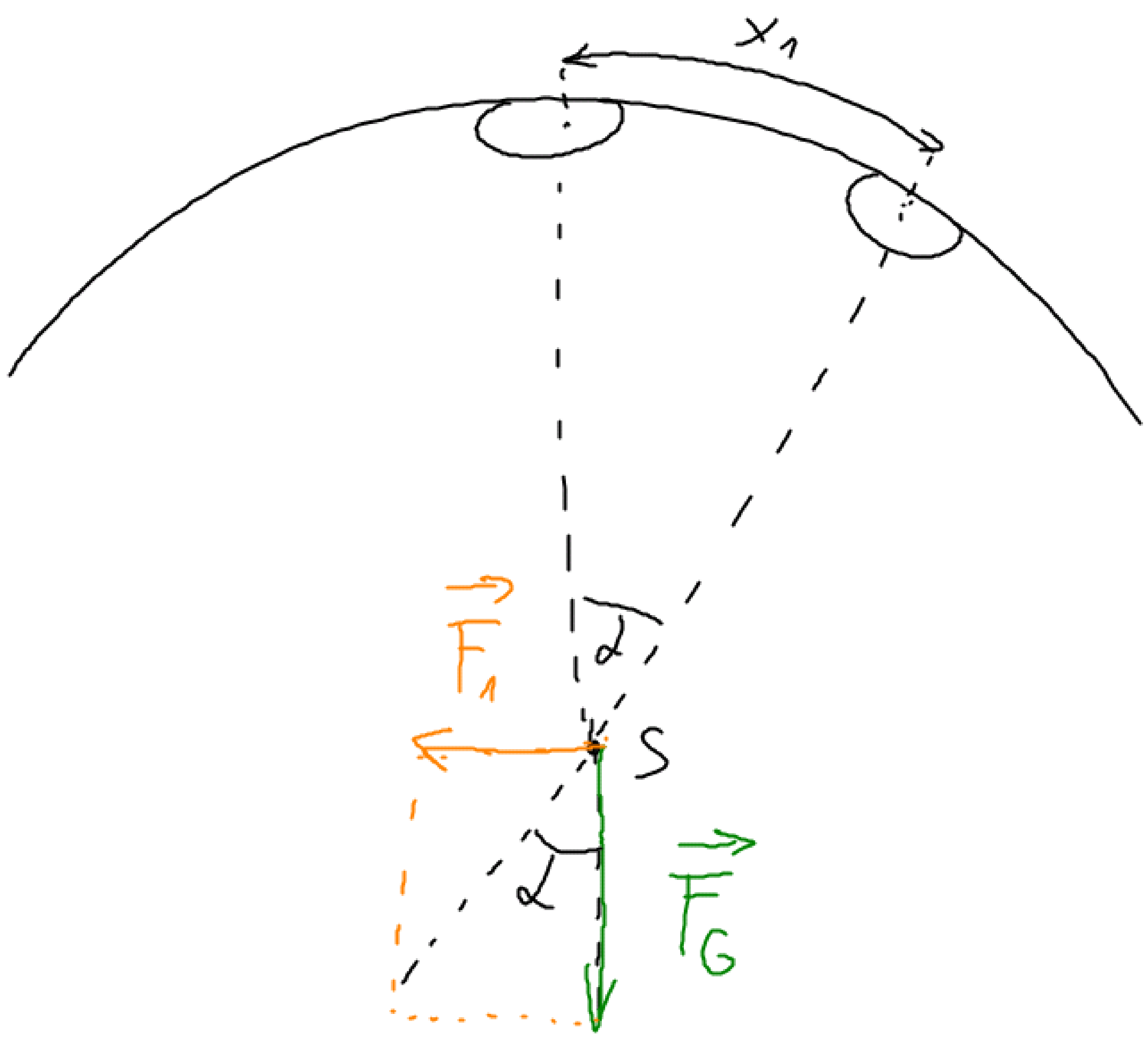




b)

$$s = \frac{1}{2} a_1 t_1^2 \Rightarrow a_1 = \frac{2s}{t_1^2}$$

$$v_1 = a_1 t_1$$



$$x_1 = R \cdot \alpha$$

$$\operatorname{tg}\alpha = \frac{F_1}{F_G} = \frac{a_1}{g}$$

$$a_1 = g \cdot \operatorname{tg}\left(\frac{x_1}{R}\right)$$

$$a_1 = 9{,}81 \cdot \operatorname{tg}\left(\frac{12}{60}\right) \text{ m}\cdot\text{s}^{-2}$$

$$\underline{\underline{a_1 = 1{,}99 \text{ m}\cdot\text{s}^{-2}}}$$

d) $a_3 = -g \operatorname{tg}\left(\frac{x_3}{R}\right)$

c) $\underline{\underline{v_{max}}} = v_{\text{na konci zrychlení}} = a_1 t_1 = g\, t_1\, tg\left(\frac{x_1}{R}\right) \doteq \underline{\underline{24\ m \cdot s^{-1}}}$

e) $a_3 = \frac{\Delta v_3}{t_3} = \frac{0 - v_{max}}{t_3}$

$\underline{t_3} = \frac{-v_{max}}{a_3} = \frac{-g\, t_1\, tg\left(\frac{x_1}{R}\right)}{-g\, tg\left(\frac{x_3}{R}\right)} = t_1 \frac{tg\left(\frac{x_1}{R}\right)}{tg\left(\frac{x_3}{R}\right)} = \underline{\underline{9{,}5\ s}}$

f) $d = s_1 + s_2 + s_3$

$d = \frac{1}{2} a_1 t_1^2 + v_{max} t_2 + v_{max} t_3 + \frac{1}{2} a_3 t_3^2 =$

$= \frac{1}{2} g\, tg\left(\frac{x_1}{R}\right) t_1^2 + g\, t_1 t_2\, tg\left(\frac{x_1}{R}\right) + g\, t_1 t_3\, tg\left(\frac{x_1}{R}\right) - \frac{1}{2} g\, tg\left(\frac{x_3}{R}\right) t_3^2$

$\underline{\underline{d = 1450\ m}}$

g) $\underline{\underline{v_p}} = \dfrac{d}{t_1 + t_2 + t_3} \doteq \underline{\underline{20\ m \cdot s^{-1}}}$

Potenciální energie v centrálním gr. poli

$\Rightarrow$ tedy $E_p \cancel{=} mgh$

g – tíhové zrychlení

tíhové pole – speciální případ centrálního pole,
kde $\vec{K} = \text{konst}$ (resp. g je konstantní)

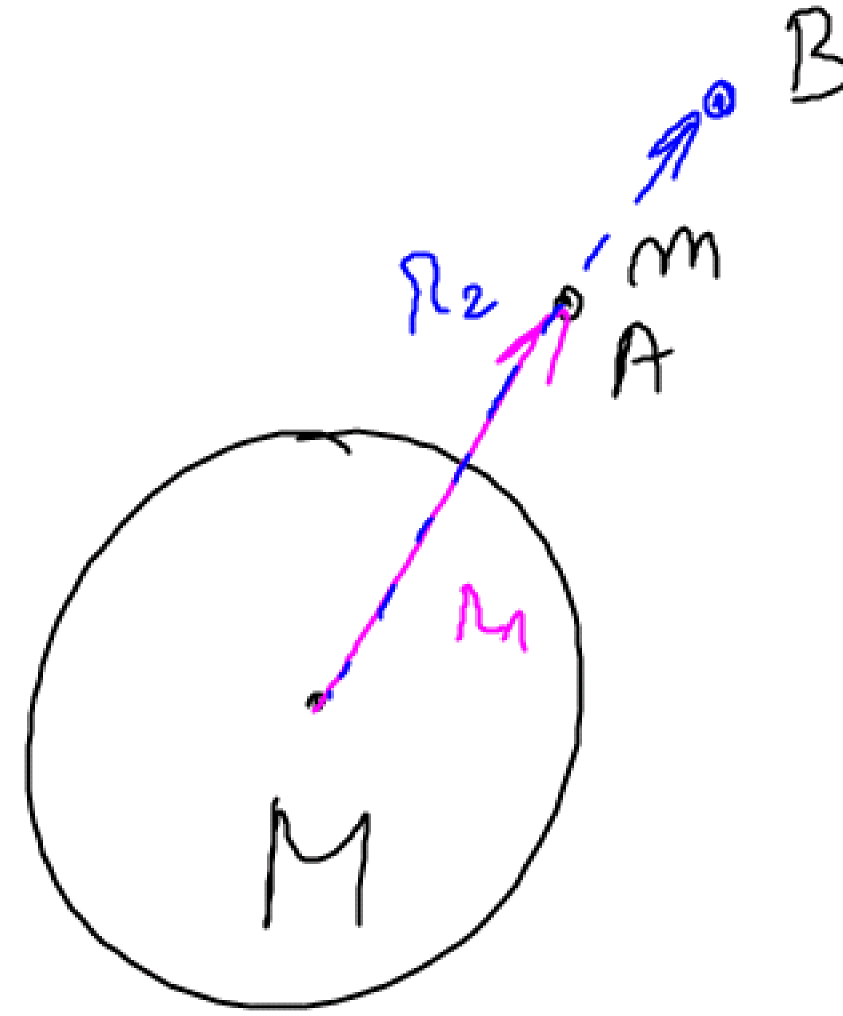
I.

$$\Delta E_p = W$$

$$\Delta E_p = F_g \cdot \Delta r$$

$$E_p = \int_{r_1}^{r_2} F_g \cdot dr$$

$$E_p = \int_{r_1}^{r_2} \frac{\varkappa M m}{r^2} dr = \varkappa M m \left[-\frac{1}{r}\right]_{r_1}^{r_2} = -\varkappa M m \left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \varkappa M m \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



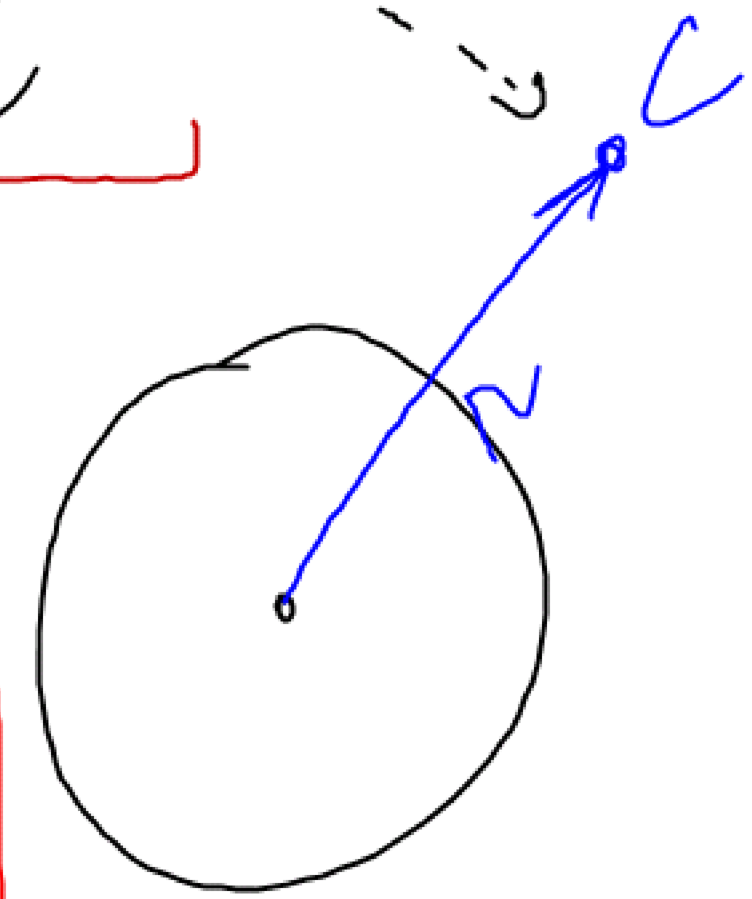
Země

přesun z A do B

předpoklad: $E_{pB} > E_{pA}$ ... B je dále od Země;

odpovídá to i fyzikální představě

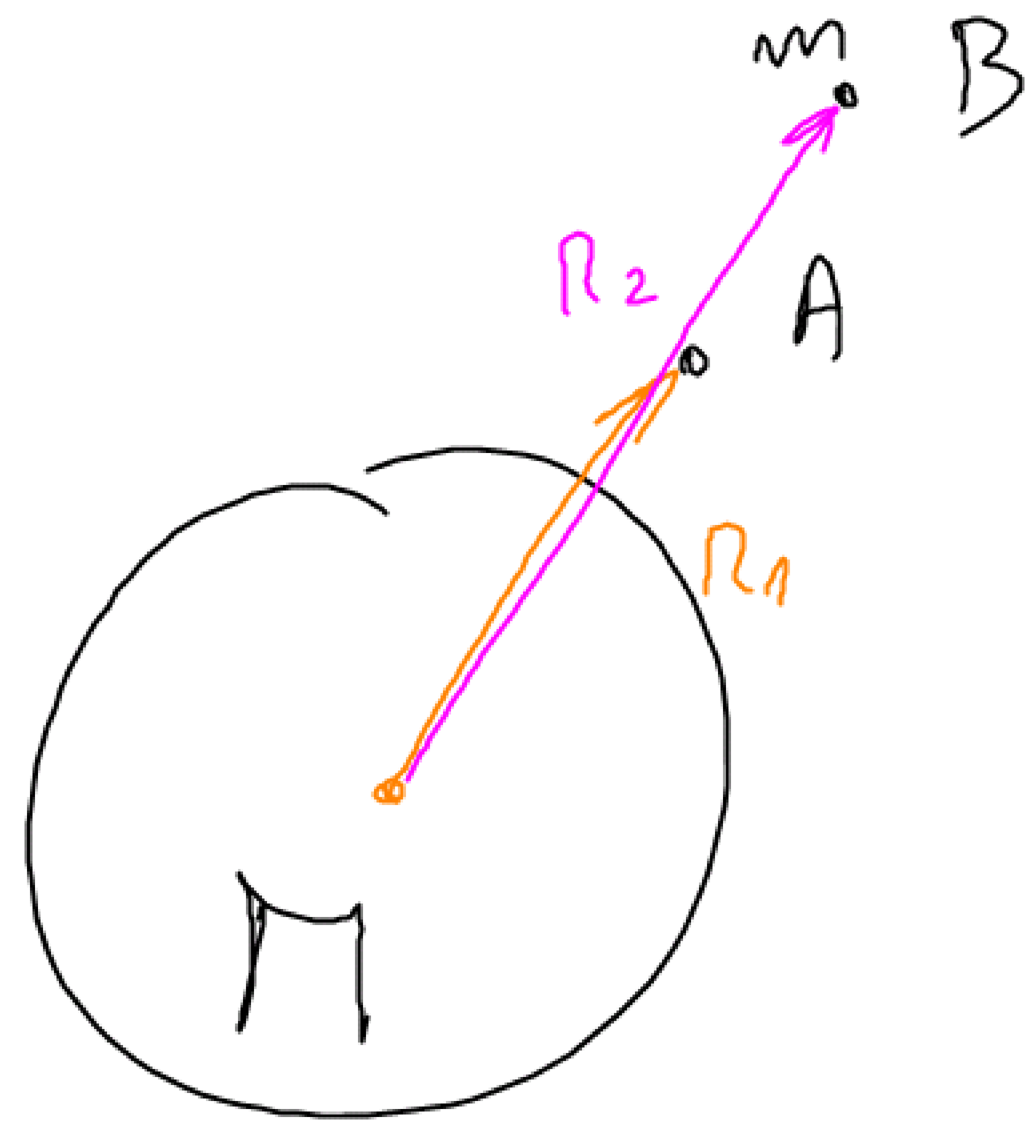
je lákavé psát: $E_p = \frac{\varkappa M m}{r}$



$r \to \infty \Rightarrow E_p \to 0$

$\Rightarrow$ náprava: $\boxed{E_p = -\frac{\varkappa M m}{r}}$

II. bez použití integrálu



$$F_{g_1} = \varkappa \frac{Mm}{r_1^2}$$

$$F_{g_2} = \varkappa \frac{Mm}{r_2^2}$$

$F = f(r)$ není lineární $\Rightarrow$

$\Rightarrow$ při hledání PRŮMĚRNÉ velikosti síly použijeme GEOMETRICKÝ PRŮMĚR

$$F_p = \sqrt{F_{g_1} \cdot F_{g_2}} = \sqrt{\varkappa \frac{Mm}{r_1^2} \cdot \varkappa \frac{Mm}{r_2^2}} = \frac{\varkappa Mm}{r_1 r_2}$$

(„průměrná síla mezi body A a B“)

$$W = F_p \cdot (r_2 - r_1) \quad \ldots \text{ práce síly } \vec{F_p} \text{ mezi body } \underline{A}, \underline{B}$$

$$W = \frac{\varkappa M m}{r_1 r_2}(r_2 - r_1) = \frac{\varkappa M m \cancel{r_2}}{r_1 \cancel{r_2}} - \frac{\varkappa M m \cancel{r_1}}{\cancel{r_1} r_2} =$$

$$= \frac{\varkappa M m}{r_1} - \frac{\varkappa M m}{r_2} = \underbrace{-\frac{\varkappa M m}{r_2}}_{E_{p_2}} - \underbrace{\left(-\frac{\varkappa M m}{r_1}\right)}_{E_{p_1}}$$

$$\Longrightarrow \boxed{E_p = -\frac{\varkappa M m}{r}}$$

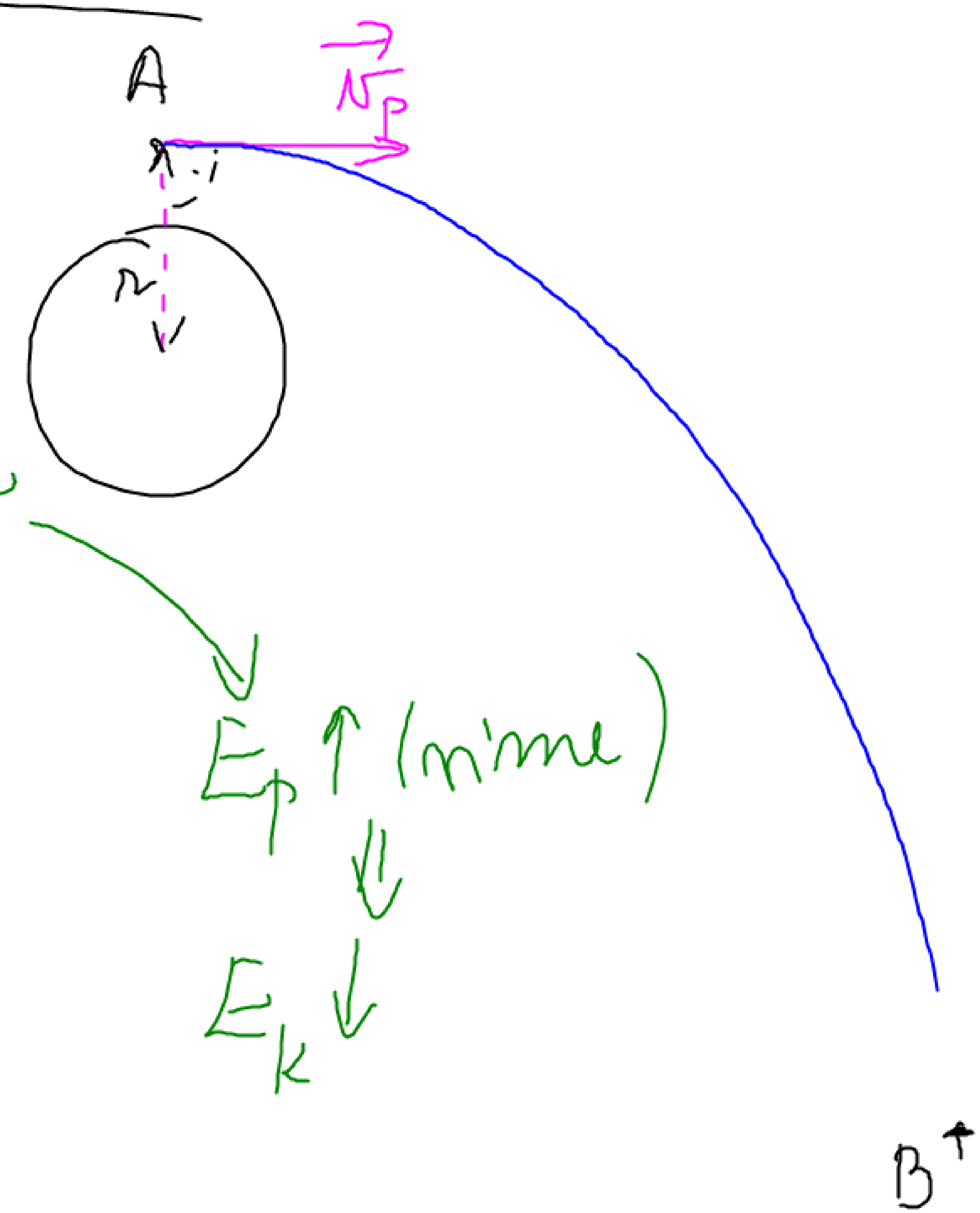
Úniková (parabolická) rychlost

ZZE: $E_{k_A} + E_{p_A} = E_{k_B} + E_{p_B}$

$$\frac{1}{2} m v_p^2 - \frac{\varkappa M m}{r} = \underbrace{0 + 0}$$

$$\boxed{v_p = \sqrt{\frac{2\varkappa M}{r}}}$$

$$v_p = \sqrt{2}\, v_k$$



$E_p \uparrow$ (míně)

$\Downarrow$

$E_k \downarrow$

FYZIKÁLNÍ OLYMPIÁDA

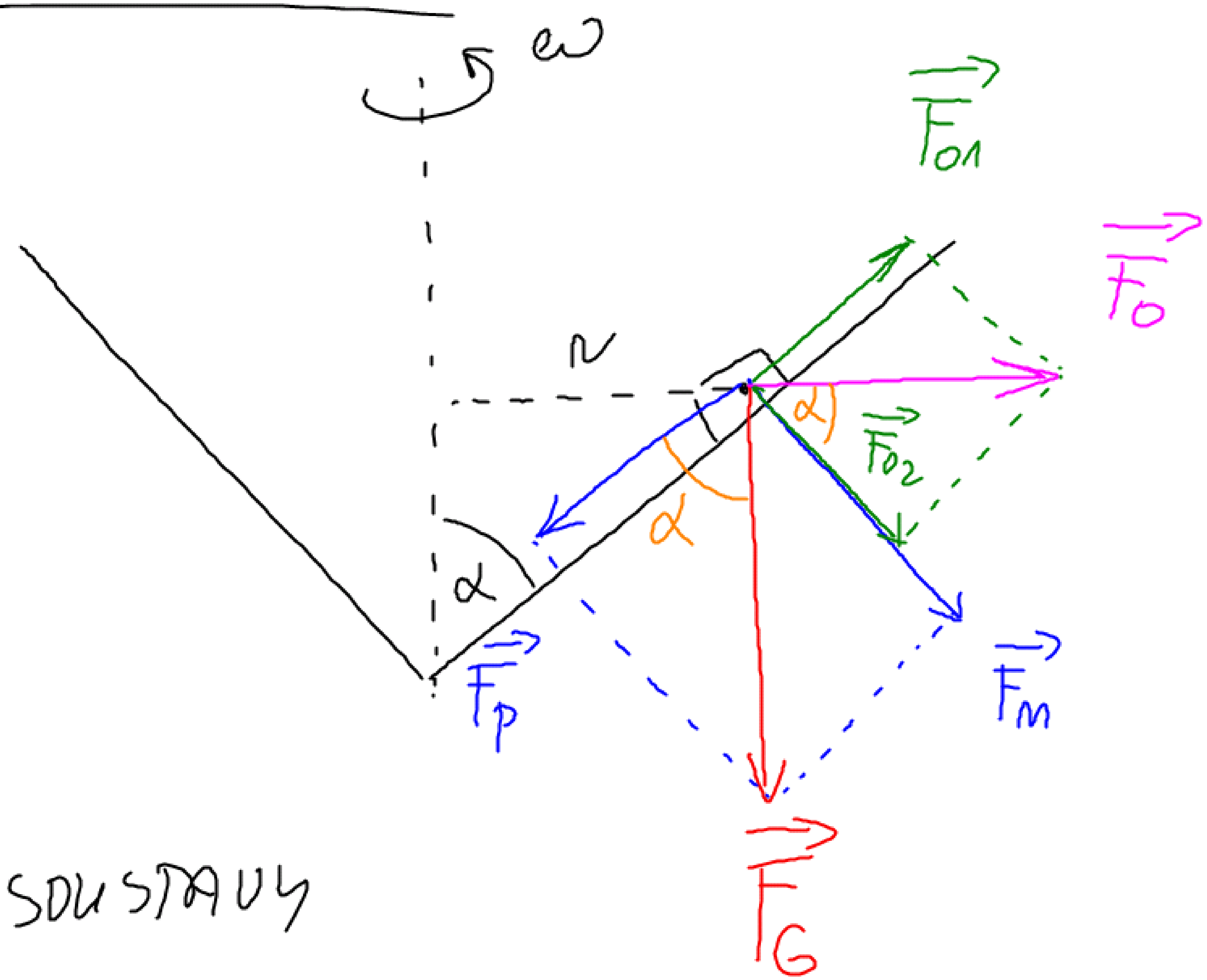
FO – 31 – 1 – C

$\alpha = 45°$

$r = 3\,\text{cm}$

$f = 0{,}2$

$\omega = ?$



z NEINERCIÁLNÍ SOUSTAVY

$\vec{F}_o$ - odstředivá síla

$\vec{F}_n$ způsobí třecí sílu $\vec{F}_{tn}$

$\vec{F}_{o2}$ způsobí třecí sílu $\vec{F}_{to2}$

a) kuličky sjíždí dolů $\Rightarrow \vec{F_O}$ tomu musí bránit

$\Rightarrow \vec{F_p}$ míří dolů, $\vec{F_{tm}}, \vec{F_{to2}}, \vec{F_{o1}}$ míří nahoru

$$F_p \leq F_{o1} + F_{tm} + F_{to2}$$

$$mg\cos\alpha \leq m\omega^2 r \sin\alpha + f mg \sin\alpha + f m\omega^2 r\cos\alpha$$

$$g(\cos\alpha - f\sin\alpha) \leq \omega^2 r(\sin\alpha + f\cos\alpha)$$

$$\omega \geq \sqrt{\frac{g}{r}\cdot\frac{\cos\alpha - f\sin\alpha}{\sin\alpha + f\cos\alpha}}$$

$$\omega \geq \sqrt{\frac{9{,}8}{0{,}03}\cdot\frac{1-0{,}2}{1+0{,}2}}\ \mathrm{s}^{-1}$$

$$\underline{\omega \geq 14{,}8\ \mathrm{s}^{-1}}$$

b) kostka jede nahoru ⇒ $\vec{F}_p$, $\vec{F}_{tm}$, $\vec{F}_{to2}$ míří dolů a $\vec{F}_{o1}$ míří nahoru

$$F_p + F_{tm} + F_{to2} \geq F_{o1}$$

$$mg\cos\alpha + f mg\sin\alpha + f m\omega^2 r\cos\alpha \geq m\omega^2 r\sin\alpha$$

$$\omega^2 r(-f\cos\alpha + \sin\alpha) \leq g(\cos\alpha + f\sin\alpha)$$

$$\omega \leq \sqrt{\frac{g}{r}\,\frac{\cos\alpha + f\sin\alpha}{\sin\alpha - f\cos\alpha}}$$

$$\omega \leq \sqrt{\frac{9{,}8}{0{,}03}\cdot\frac{1+0{,}2}{1-0{,}2}}\ \mathrm{s}^{-1}$$

$$\underline{\omega \leq 22{,}1\ \mathrm{s}^{-1}}$$

$$\Rightarrow \omega \in \langle 14{,}8 \, ; \, 22{,}1 \rangle \, s^{-1}$$

FO – 57 – 1 – A

$m = 20\,g$

$v_0 = 150\,m\cdot s^{-1}$

$M = 500\,g$

$v = 250\,m\cdot s^{-1}$

---

a) $\Delta U = ?$

b) $u = ?$

c) $u_{max} = ?$

a) $\Delta U \sim F_{DRZENA} \quad \sim F_{DRZENA}$ NEZÁVISÍ

na $\underline{\vec{v}_{střel}}$ $\Rightarrow$ lze počítat

pro $\underline{v_0}$ $\Rightarrow$ nepřesný

naráz (neboť $\underline{v_0}$ je minimální)

$$ZZH:\ m v_0 = (M+m)\, v_1 \quad (1)$$

$$ZZE:\ \frac{1}{2} m v_0^2 = \frac{1}{2}(M+m) v_1^2 + \Delta U \quad (2)$$

$$(1) \Rightarrow v_1 = v_0 \frac{m}{M+m}$$

$$\text{do } (2):\ \frac{1}{2} m v_0^2 = \frac{1}{2}\cancel{(M+m)}\, v_0^2 \frac{m^2}{(M+m)^{\cancel{2}}} + \Delta U$$

$$\Delta U = \frac{1}{2} v_0^2 m \left(1 - \frac{m}{M+m}\right) = \frac{1}{2} v_0^2 \frac{Mm}{M+m}$$

$$\underline{\underline{\Delta U}} = \frac{1}{2}\cdot 150^2 \frac{0{,}5\cdot 2\cdot 10^{-2}}{0{,}52}\ \mathrm{J} = \underline{\underline{216\ \mathrm{J}}}$$

b) ZZH: $mv = mv_1 + Mu \qquad (3)$

ZZE: $\frac{1}{2}mv^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}Mu^2 + \Delta U \qquad (4)$

$$(3) \Rightarrow \quad v_2 = \frac{mv - Mu}{m}$$

$$\text{do } (4):\ \cancel{\tfrac{1}{2}}mv^2 = \cancel{\tfrac{1}{2}}\cancel{m}\frac{(mv-Mu)^2}{m^{\cancel{2}}} + \cancel{\tfrac{1}{2}}Mu^2 + \cancel{\tfrac{1}{2}}v_0^2\frac{Mm}{M+m} \quad \Big/ \cdot m(M+m)$$

$$m^2(M+m)v^2 = (M+m)(m^2v^2 - 2mMvu + M^2u^2) + mM(M+m)u^2 + Mm^2v_0^2$$

$$0 = -2\cancel{M} m (M+m) v u + (M+m) M^{\cancel{2}} u^2 + m \cancel{M} (M+m) u^2 +$$
$$+ \cancel{M} m^2 v_0^2$$

$$u^2 (M+m)^2 - 2m(M+m) v u + m^2 v_0^2 = 0$$

$$u_{1,2} = \frac{2m(M+m)v \pm \sqrt{4m^2(M+m)^2 v^2 - 4m^2 v_0^2 (M+m)^2}}{2(M+m)^2} =$$

$$= \frac{\cancel{2} m \cancel{(M+m)} v \pm \cancel{2} m \cancel{(M+m)} \sqrt{v^2 - v_0^2}}{\cancel{2}(M+m)^{\cancel{2}}} =$$

$$= \frac{m}{M+m}\left(v \pm \sqrt{v^2 - v_0^2} \right) > 0$$

$v \uparrow \Rightarrow$ čas interakce střely a vozíku klesá $\Rightarrow$ změna

Hmotnosti předaná vozíku KLESÁ $\Rightarrow$ velikost rychlosti vozíku bude menší $\Rightarrow$ z $u_{1,2}$ vyhovuje menší kořen : $u = \frac{m}{M+m}\left(v - \sqrt{v^2 - v_0^2}\right)$

$$u = \frac{20}{520}\left(250 - \sqrt{250^2 - 150^2}\right) \text{ m}\cdot\text{s}^{-1}$$

$$u = 1{,}9 \text{ m}\cdot\text{s}^{-1}$$

c) u_{max} nastane pro minimální velikost rychlosti střely $\Rightarrow$ $u_{max} = \frac{m}{M+m}\left(v_0 - \sqrt{v_0^2 - v_0^2}\right) = \frac{m v_0}{M+m}$

$$u_{max} = \frac{20}{520}\cdot 150 \text{ m}\cdot\text{s}^{-1} = 5{,}8 \text{ m}\cdot\text{s}^{-1}$$

FO - 37 - Slovensko

$h = 760\ km$

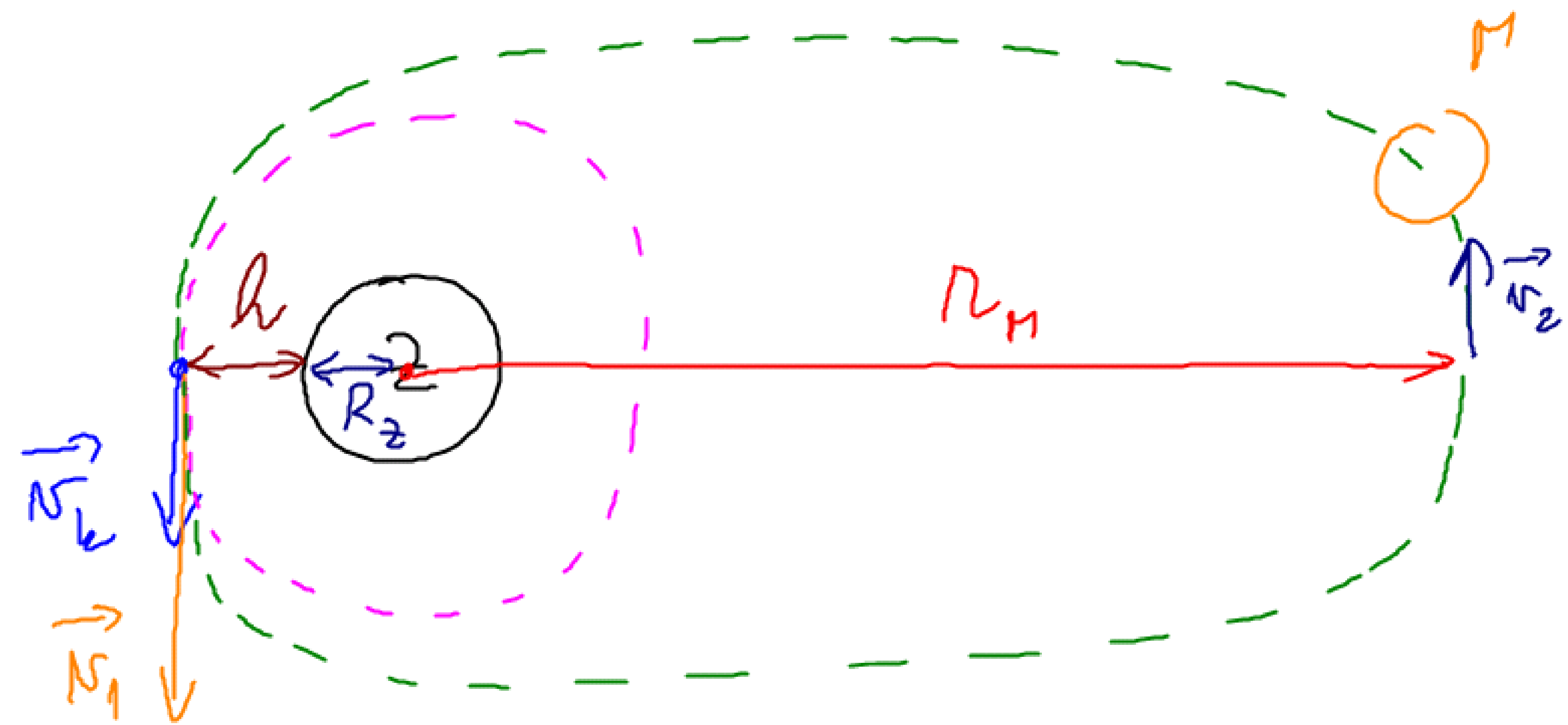
$M_Z = 6 \cdot 10^{24}\ kg$

$R_Z = 6{,}38 \cdot 10^{6}\ m$

$r_M = 3{,}85 \cdot 10^{8}\ m$

$\Delta v = ?$

$t = ?$



$$F_g = F_d$$

$$\kappa \frac{M_Z m}{r^2} = m \frac{v_k^2}{r}$$

$$v = \sqrt{\frac{\kappa M_Z}{r}} = \sqrt{\frac{\kappa M_Z}{R_Z + h}}$$

$$v_k = \sqrt{\frac{6{,}67\cdot 10^{-11}\cdot 6\cdot 10^{24}}{6{,}38\cdot 10^{6}+0{,}76\cdot 10^{6}}}\ \mathrm{m\cdot s^{-1}} = 7{,}49\ \mathrm{km\cdot s^{-1}}$$

$$\text{ZZE: } E_{kmZ} + E_{pmZ} = E_{kmM} + E_{pmM}$$

$$\frac{1}{2}\cancel{m}v_1^2 - \frac{\varkappa M\cancel{m}}{R_Z+h} = \frac{1}{2}\cancel{m}v_2^2 - \frac{\varkappa M\cancel{m}}{r_M}$$

$$\text{ZZMH: } m\cdot v_1\cdot(R_Z+h) = m\cdot v_2\cdot r_M \Rightarrow v_2 = v_1\frac{R_Z+h}{r_M}$$

$$v_1^2 - \frac{2\varkappa M}{R_Z+h} = v_1^2\frac{(R_Z+h)^2}{r_M^2} - \frac{2\varkappa M}{r_M}$$

$$v_1^2\left(1-\frac{(R_Z+h)^2}{r_M^2}\right) = 2\varkappa M\left(\frac{1}{R_Z+h}-\frac{1}{r_M}\right)$$

$$v_1^2 \frac{\cancel{(r_M - R_Z - h)}(r_M + R_Z + h)}{r_M^{\cancel{2}}} = 2\varkappa M \frac{\cancel{r_M - R_Z - h}}{(R_Z + h)\cancel{r_M}}$$

$$v_1 = \sqrt{2\varkappa M \frac{r_M}{(r_M + R_Z + h)(R_Z + h)}}$$

$$v_1 = 10{,}49 \ \mathrm{km \cdot s^{-1}}$$

$$\underline{\underline{\Delta v}} = v_1 - v_k = (10{,}49 - 7{,}49)\ \mathrm{km \cdot s^{-1}} = \underline{\underline{3\ \mathrm{km \cdot s^{-1}}}}$$

$$3.\ KZ: \left(\frac{T_E}{T_k}\right)^2 = \left(\frac{\frac{R_M + R_Z + h}{2}}{R_Z + h}\right)^3$$

$$T_E = T_k \cdot \sqrt{\left(\frac{R_M + R_Z + h}{2(R_Z + h)}\right)^3}$$

$$T_k = \frac{2\pi (R_Z + h)}{v_k} = 2\pi (R_Z + h) \cdot \sqrt{\frac{R_Z + h}{\varkappa M_Z}}$$

$$T_E = 2\pi \sqrt{\frac{(R_M + R_Z + h)^3}{8(R_Z + h)} \cdot \frac{(R_Z + h)}{\varkappa M_Z}} = \pi \sqrt{\frac{(R_M + R_Z + h)^3}{2 \varkappa M_Z}}$$

$$\underline{\underline{t}} = \frac{T_E}{2} = \frac{\pi}{2} \sqrt{\frac{(R_M + R_Z + h)^3}{2 \varkappa M_Z}} = 4{,}31 \cdot 10^5\,\mathrm{s} \doteq \underline{\underline{5\ \text{dní}}}$$

FO-57-1-C

$m_1 = 0,3\,kg$

$m_2 = 0,8\,kg$

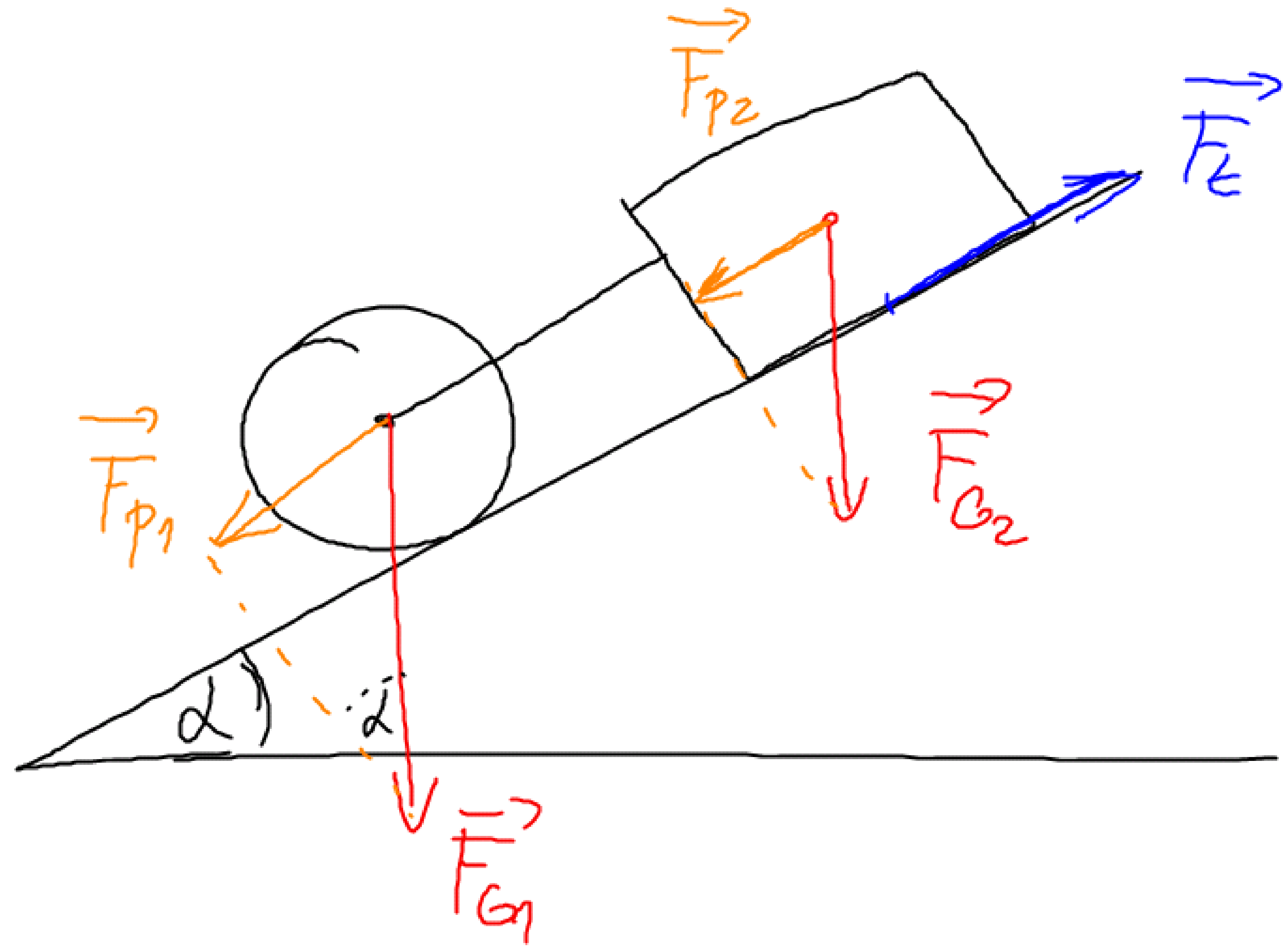
$f = 0,4$

---

a) $\alpha_1 = ?$ $\quad v = konst$

b) $\alpha_2 = ?$

c) $a_x = ?$ $\quad \alpha = 12°$



a) $v = konst \xRightarrow{1.NZ} F = 0$

$$F_{p_1} + F_{p_2} = F_t$$

$$m_1 g \sin\alpha_1 + m_2 g \sin\alpha_1 = m_2 g f \cos\alpha_1$$

$$(m_1 + m_2)\sin\alpha_1 = m_2 f \cos\alpha_1$$

$$\frac{\sin\alpha_1}{\cos\alpha_1} = \frac{m_2 f}{m_1 + m_2}$$

$$\mathrm{tg}\,\alpha_1 = \frac{m_2 f}{m_1 + m_2}$$

$$\alpha_1 = 16{,}2^\circ$$

b) $F_{tahla} = 0 \Rightarrow a_1 = a_2$

válec: $$\underbrace{m_1 a_1}_{\text{posuvný pohyb težiště}} + \underbrace{\frac{J\varepsilon}{r}}_{\text{rotační pohyb}} = m_1 g \sin\alpha_2$$

$$\cancel{m_1} a_1 + \frac{\cancel{m_1} \cancel{r^2}}{2} \cdot \frac{a_1}{\cancel{r}} \cdot \frac{1}{\cancel{r}} = \cancel{m_1} g \sin\alpha_2$$

$$\frac{3}{2} a_1 = g \sin\alpha_2$$

$$a_1 = \frac{2}{3} g \sin\alpha_2$$

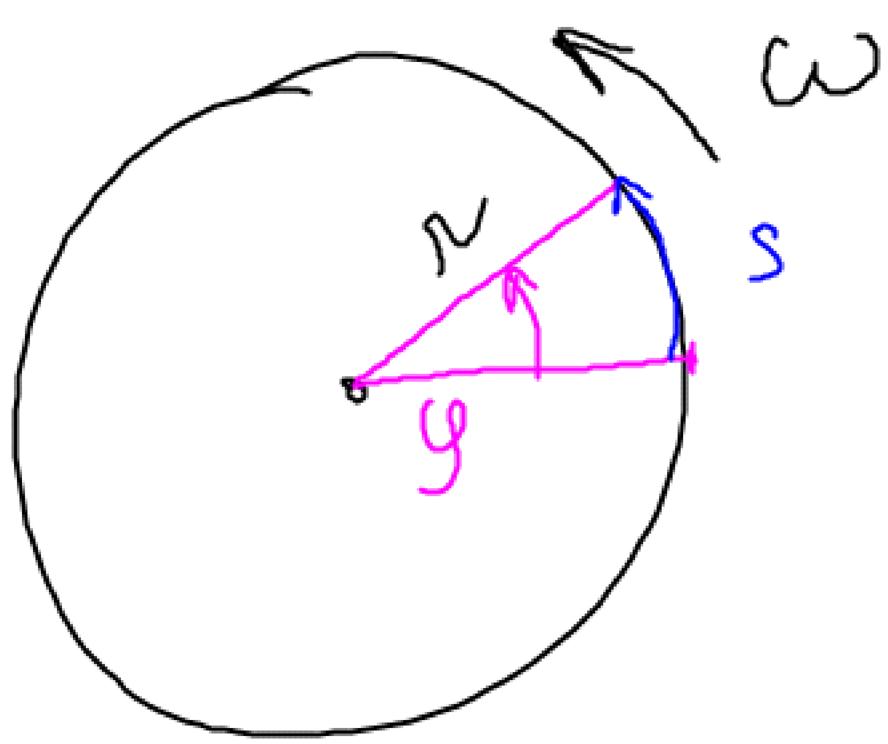
odkud $\frac{J\varepsilon}{r} = ?$

2. VI: $M = \frac{dL}{dt} = \frac{d(J\omega)}{dt} = J\frac{d\omega}{dt} = J\varepsilon$ ... úhlové zrychlení!

$M = F \cdot r$

$M = J\varepsilon$

$\Rightarrow F = \frac{J\varepsilon}{r}$



$s = \frac{1}{2} a t^2$

$\varphi = \frac{1}{2} \varepsilon \cdot t^2$

$\frac{s}{\varphi} = \frac{a}{\varepsilon}$

$r = \frac{a}{\varepsilon} \Rightarrow \varepsilon = \frac{a}{r}$

kvádra:

$$F_{p_2} - F_t = m_2 a_2$$

$$m_2 g \sin\alpha_2 - f m_2 g \cos\alpha_2 = m_2 a_2$$

$$a_2 = g(\sin\alpha_2 - f\cos\alpha_2)$$

$$a_1 = a_2 \Rightarrow \frac{2}{3}\cancel{g}\sin\alpha_2 = \cancel{g}(\sin\alpha_2 - f\cos\alpha_2)$$

$$-\frac{1}{3}\sin\alpha_2 = -f\cos\alpha_2$$

$$\mathrm{tg}\,\alpha_2 = 3f$$

$$\alpha_2 \doteq 50^\circ$$

c)

$$F_{p_1} + F_{p_2} - F_t = m_1 a_x + \frac{J\varepsilon}{r} + m_2 a_x$$

$$m_1 g \sin\alpha + m_2 g \sin\alpha - f m_2 g \cos\alpha = m_1 a_x + \frac{m_1 \cancel{r^2}}{2 \cancel{r}} \cdot \frac{a_x}{\cancel{r}} + m_2 a_x$$

$$g\left(m_1 \sin\alpha + m_2 \sin\alpha - f m_2 \cos\alpha\right) = a_x\left(\frac{3}{2} m_1 + m_2\right)$$

$$\underline{\underline{a_x = 2g\,\frac{(m_1 + m_2)\sin\alpha - f m_2 \cos\alpha}{3m_1 + 2m_2}}}$$

FO - 57 - 11 - C

$\alpha = 10°$

$m = 150\ kg$

$R = 70\ cm$

$r = 40\ cm$

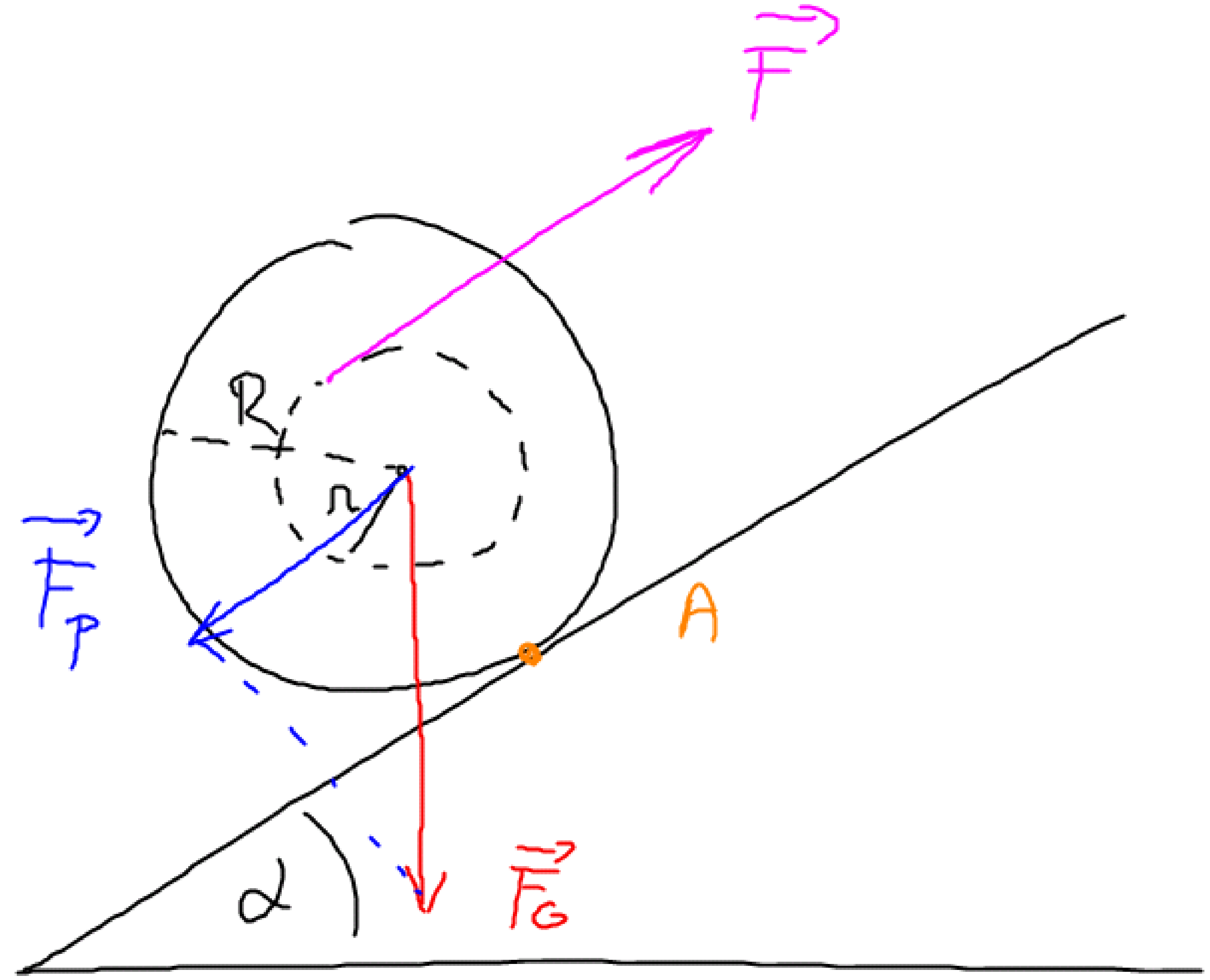
$t = 5\ s$

---

a) $v = ?$

b) $F = ?$

c) $P = ?$



a) za čas t: $s = 2\pi R + 2\pi r$

$$v = \frac{s}{t} = \frac{2\pi(R+r)}{t}$$

$$v = \frac{6{,}28 \cdot (0{,}7 + 0{,}4)}{5}\ m \cdot s^{-1}$$

$$\underline{\underline{v = 1{,}4\ m \cdot s^{-1}}}$$

b) momentová věta k $\underline{A}$: $F_p \cdot R = F(R+r)$

$$mg \sin\alpha \cdot R = F \cdot (R+r)$$

$$F = \frac{R}{R+r} \cdot mg \sin\alpha$$

$$F = \frac{0{,}7}{0{,}7+0{,}5} \cdot 150 \cdot 9{,}8 \cdot \sin 10° \text{ N}$$

$$\underline{\underline{F \doteq 162 \text{ N}}}$$

c) $P = F \cdot v$

$$\underline{\underline{P}} = \frac{R\, mg \sin\alpha}{R+r} \cdot \frac{2\pi(R+r)}{t} = \frac{2\pi R\, mg \sin\alpha}{t} \doteq \underline{\underline{224 \text{ W}}}$$

FO - 40 - 1 - A

$m = 3\,kg$

$r = 0{,}1\,m$

$h = 0{,}2\,m$

$\omega = 300\,rad \cdot s^{-1}$

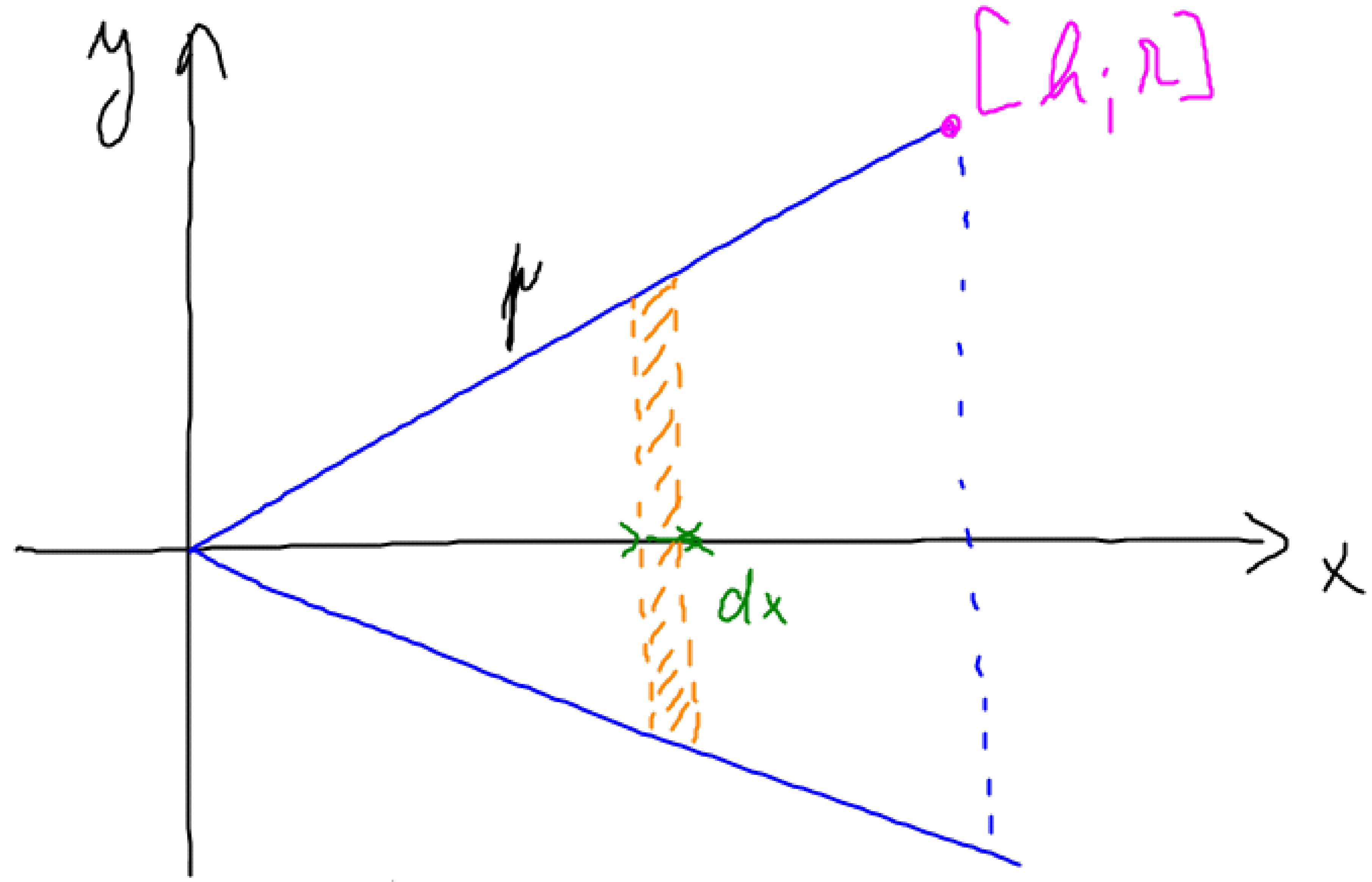
---

a) těžiště $= ?$

b) $J = ?$

c) $\Omega = ?$

a)



$p: y = kx$

$r = k \cdot h \Rightarrow k = \frac{r}{h} \Rightarrow p: y = \frac{r}{h} x$

$x \in \langle 0; h \rangle$

$$x_T = \frac{1}{m}\int_0^{m_c} x\,dm \qquad dm = \rho\,dV = \rho\cdot\left(\frac{r}{h}x\right)^2\cdot\pi\cdot dx$$

$$x_T = \frac{1}{m}\int_0^h x\cdot\rho\,\frac{r^2}{h^2}x^2\pi\,dx \qquad m = \rho\cdot\frac{1}{3}\pi r^2 h \quad (1)$$

$$x_T = \frac{3}{\cancel{\pi}\cancel{\rho}\cancel{r^2}h}\cdot\frac{\cancel{\rho}\cancel{r^2}\cancel{\pi}}{h^2}\int_0^h x^3\,dx = \frac{3}{h^3}\cdot\left[\frac{x^4}{4}\right]_0^h =$$

$$= \frac{3}{h^3}\left(\frac{h^4}{4}-0\right) = \underline{\underline{\frac{3}{4}h}} \qquad T = \left[\frac{3}{4}h;\ 0;\ 0\right] = [0{,}15;\ 0;\ 0]\,m$$

b)

$$\underline{\underline{J}} = \int_0^{m_c} \underbrace{\frac{1}{2} y^2 \, dm}_{J \text{ válečku}} = \frac{1}{2} \int_0^h \left(\frac{r}{h} x\right)^2 \cdot \rho \left(\frac{r}{h} x\right)^2 \pi \, dx =$$

$$= \frac{1}{2} \frac{r^2}{h^2} \rho \frac{r^2}{h^2} \pi \int_0^h x^4 \, dx = \frac{1}{2} \frac{r^4}{h^4} \rho \pi \left[\frac{x^5}{5}\right]_0^h =$$

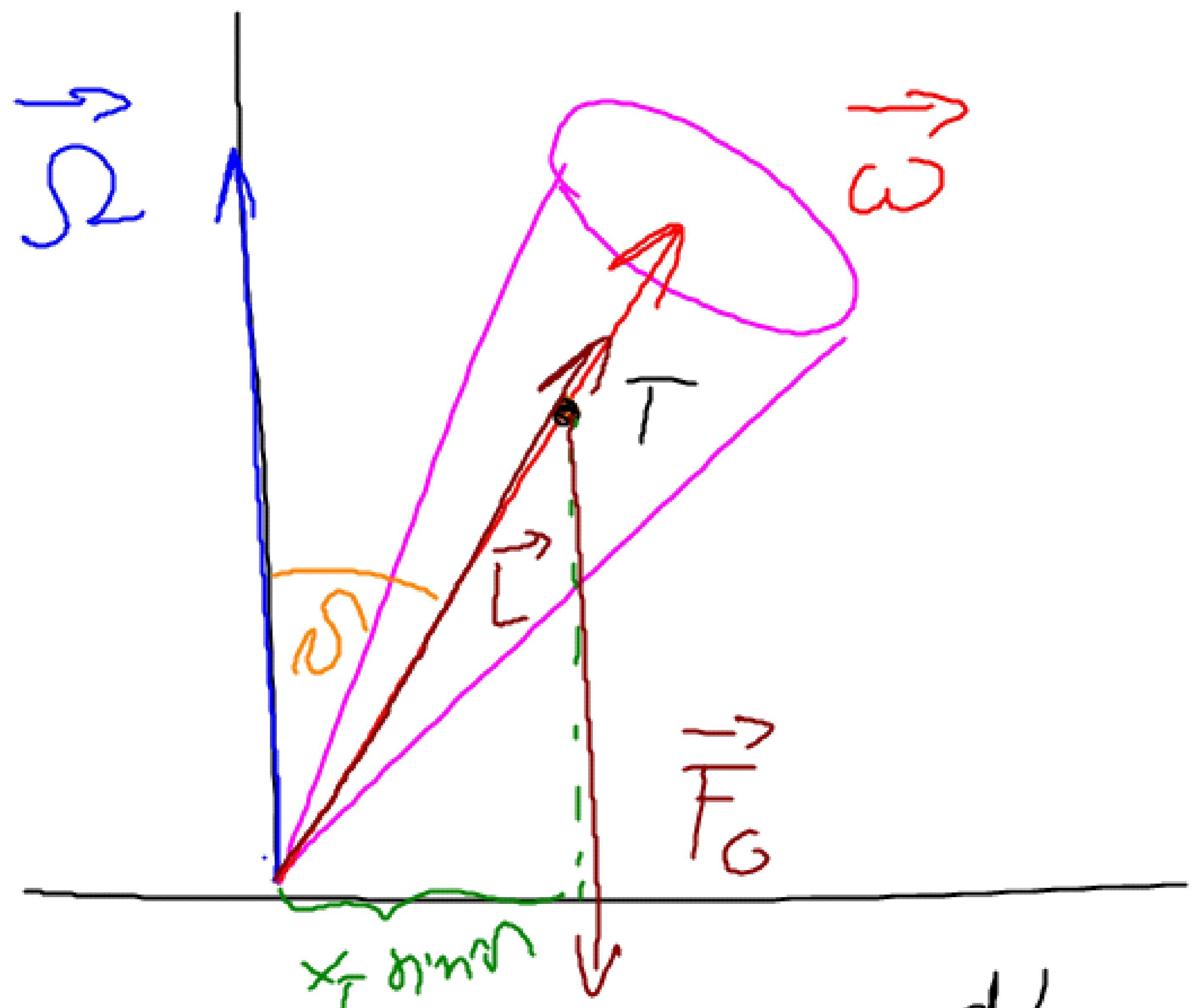
$$= \frac{1}{2} \frac{r^4}{h^4} \rho \pi \left(\frac{h^5}{5} - 0\right) = \frac{1}{10} r^4 h \rho \pi = \frac{1}{10} \cdot \cancel{r^4}^{\,2} \cancel{h} \cancel{\pi} \; \frac{3m}{\cancel{\pi} \cancel{r^2} \cancel{h}} =$$

$$(1) \Rightarrow \rho = \frac{3m}{\pi r^2 h}$$

$$= \underline{\underline{\frac{3}{10} m r^2}} =$$

$$= \frac{3}{10} \cdot 3 \cdot 0{,}1^2 \ \mathrm{kg\,m^2} = \underline{\underline{9 \cdot 10^{-3} \ \mathrm{kg \cdot m^2}}}$$

c)



$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{M} = \vec{\Omega} \times \vec{L}$$

2. impulsová věta: $M = \frac{dL}{dt} \quad \wedge \quad L = J \cdot \omega$

koncový bod $\vec{L}$ se pohybuje po kruhu s poloměrem

$$L \sin\vartheta = J\omega \sin\vartheta$$

$$M = \Omega L \sin\vartheta$$

$$mg\, x_T \sin\vartheta = \Omega J \omega \sin\vartheta$$

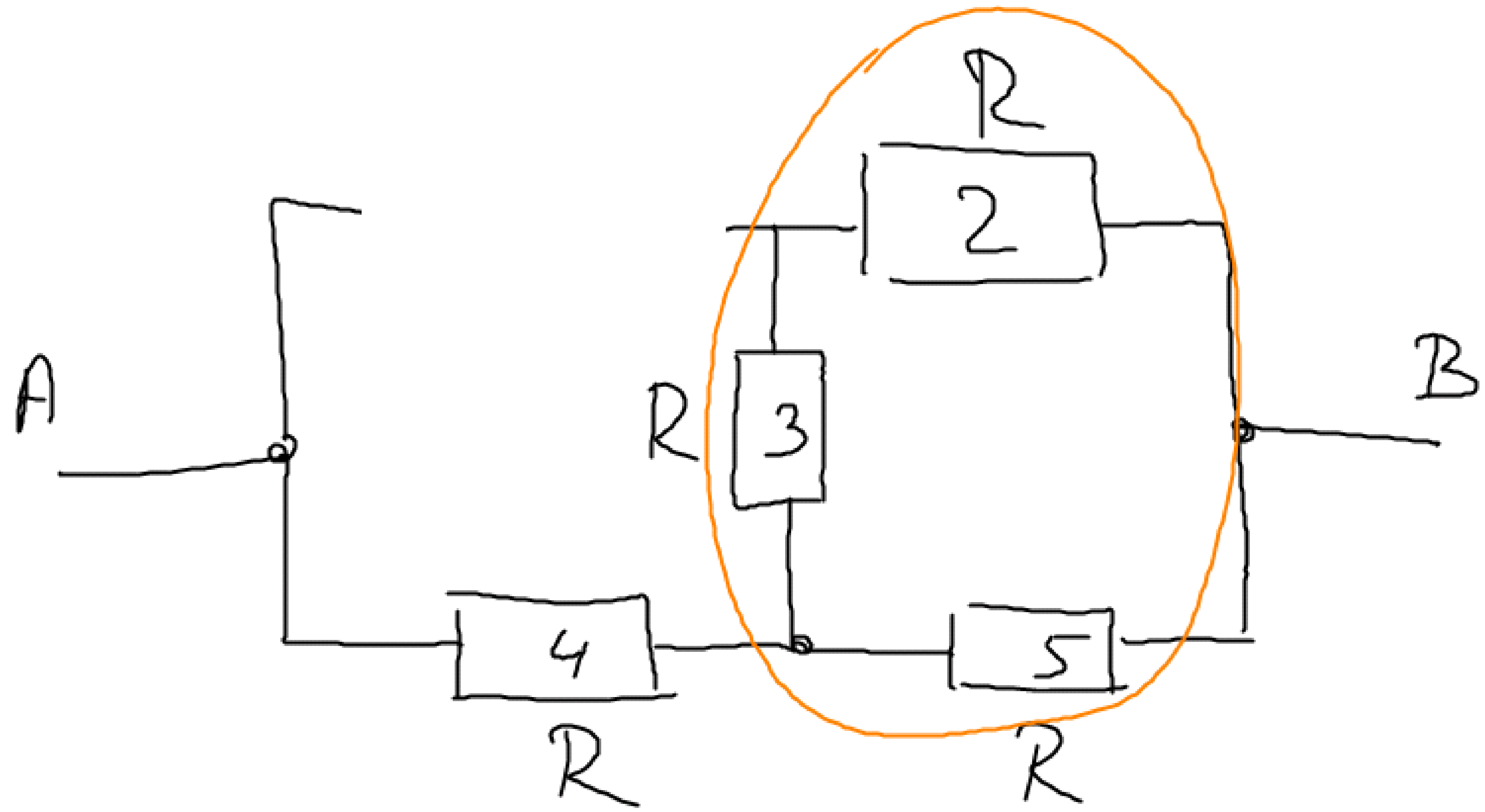
$$\Omega = \frac{mg \frac{3}{4} h}{\frac{3}{10} m r^2 \omega}$$

$$\boxed{\Omega = \frac{5gh}{2 r^2 \omega}}$$

FO – 54 – 1 – B

a)

přepálí se 1

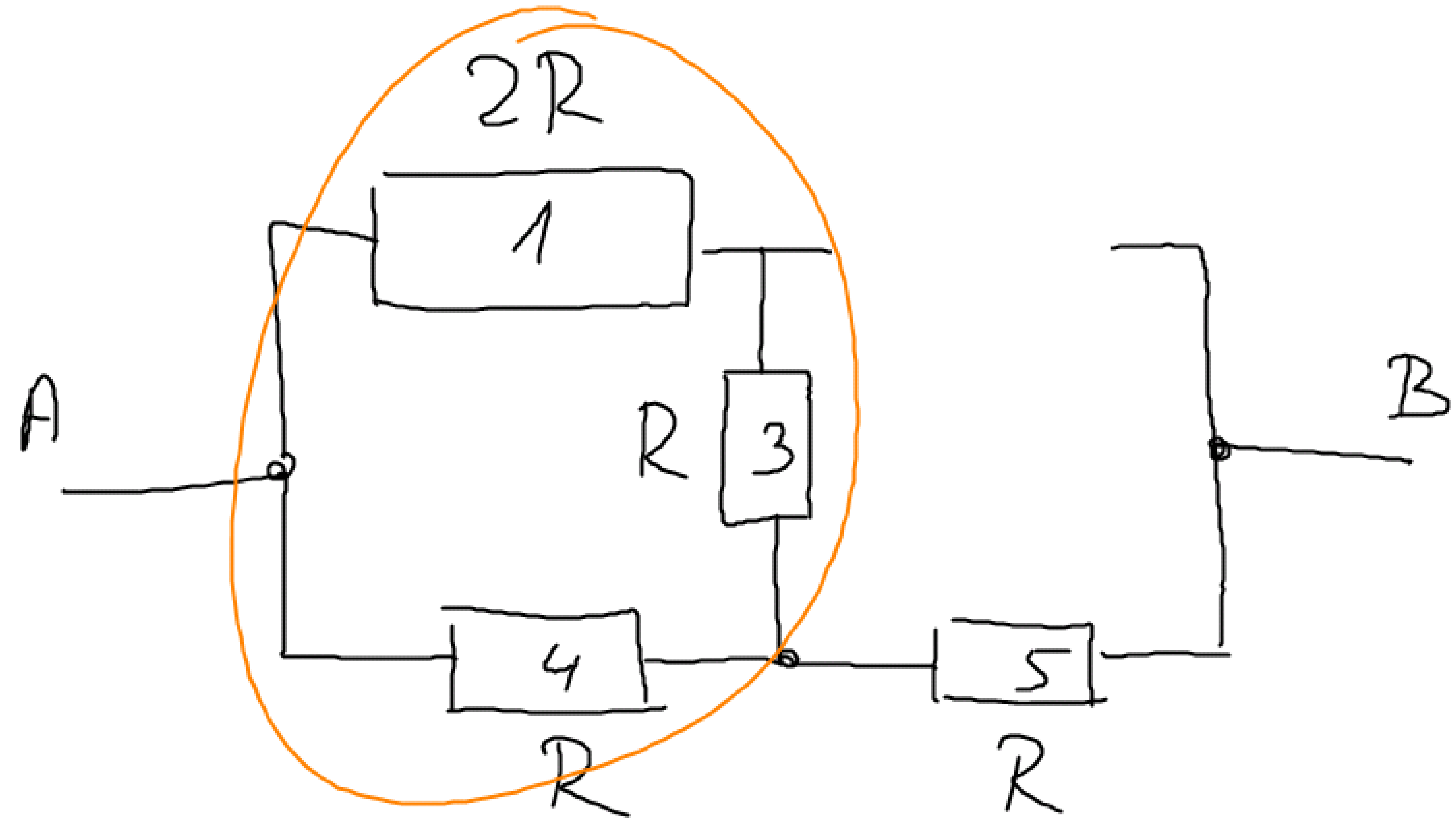


$$\frac{1}{2R} + \frac{1}{R} = \frac{1}{R_{p1}}$$

$$R_{p1} = \frac{2R}{3}$$

$$R_1 = R + \frac{2R}{3} = \frac{5}{3}R$$

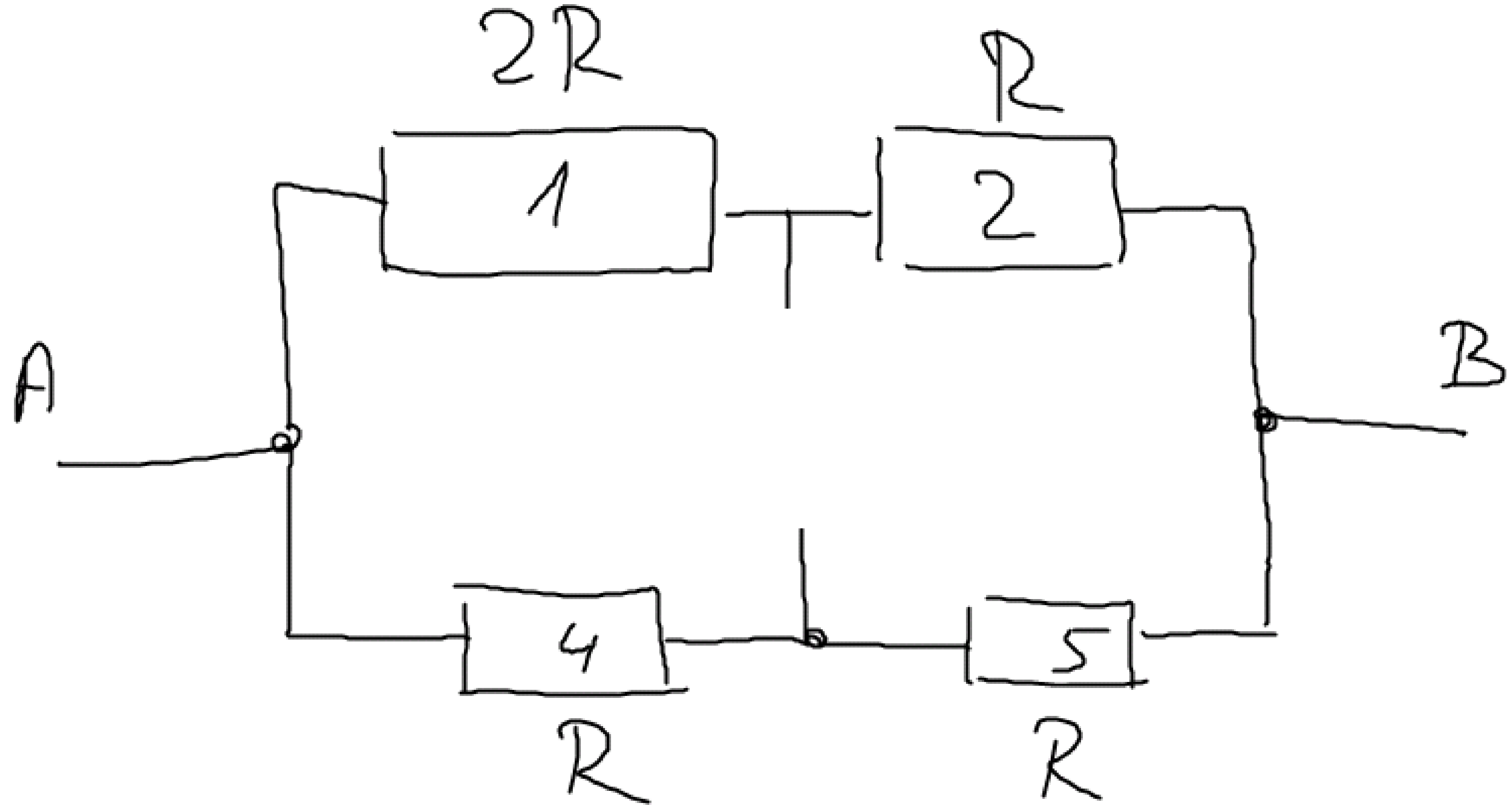
přepálí se $\underline{2}$



$$R_{p2} = \frac{3R \cdot R}{3R + R} = \frac{3}{4}R$$

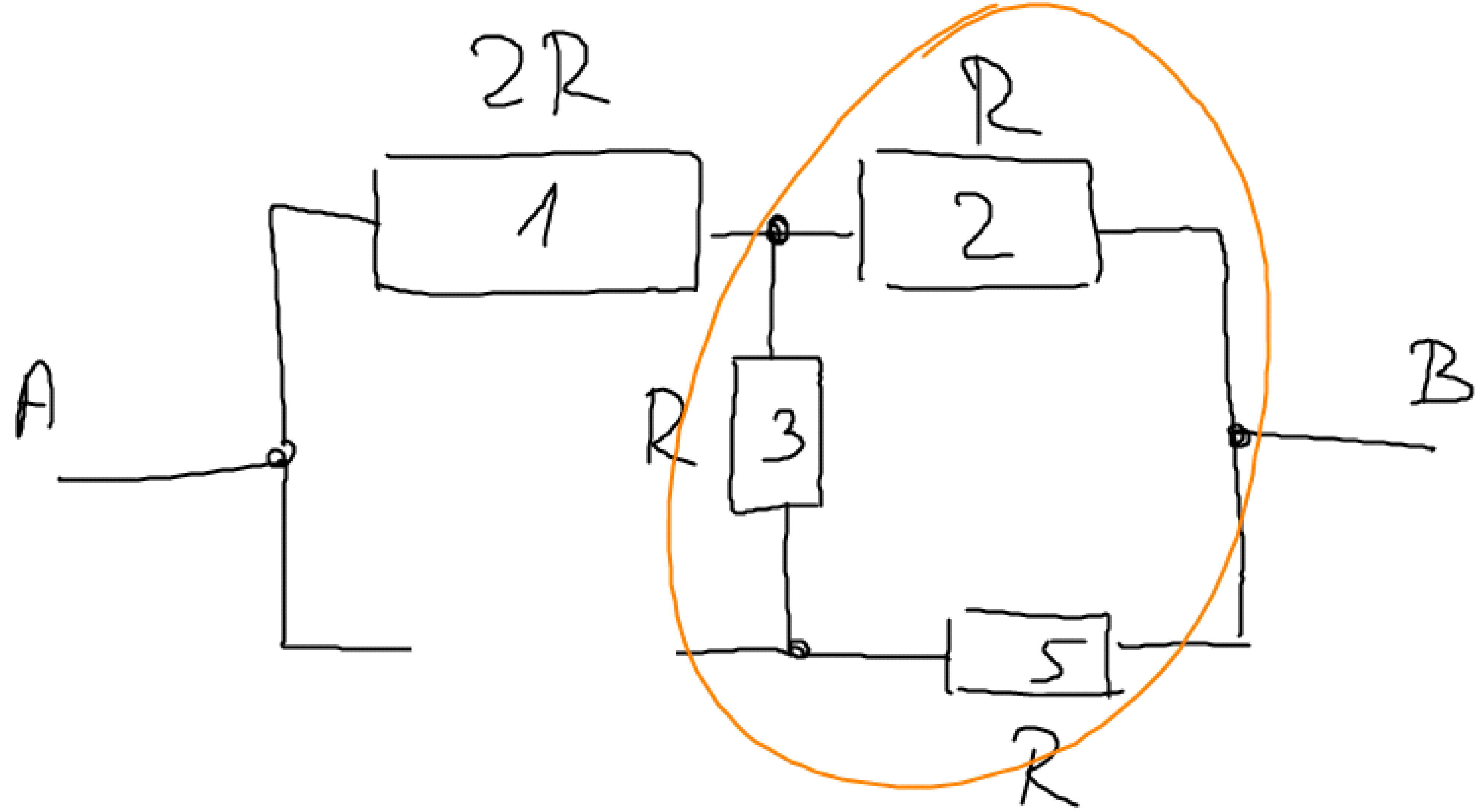
$$R_2 = \frac{3}{4}R + R = \frac{7}{4}R$$

příklad 3



$$R_3 = \frac{3R \cdot 2R}{3R + 2R} = \frac{6}{5} R$$

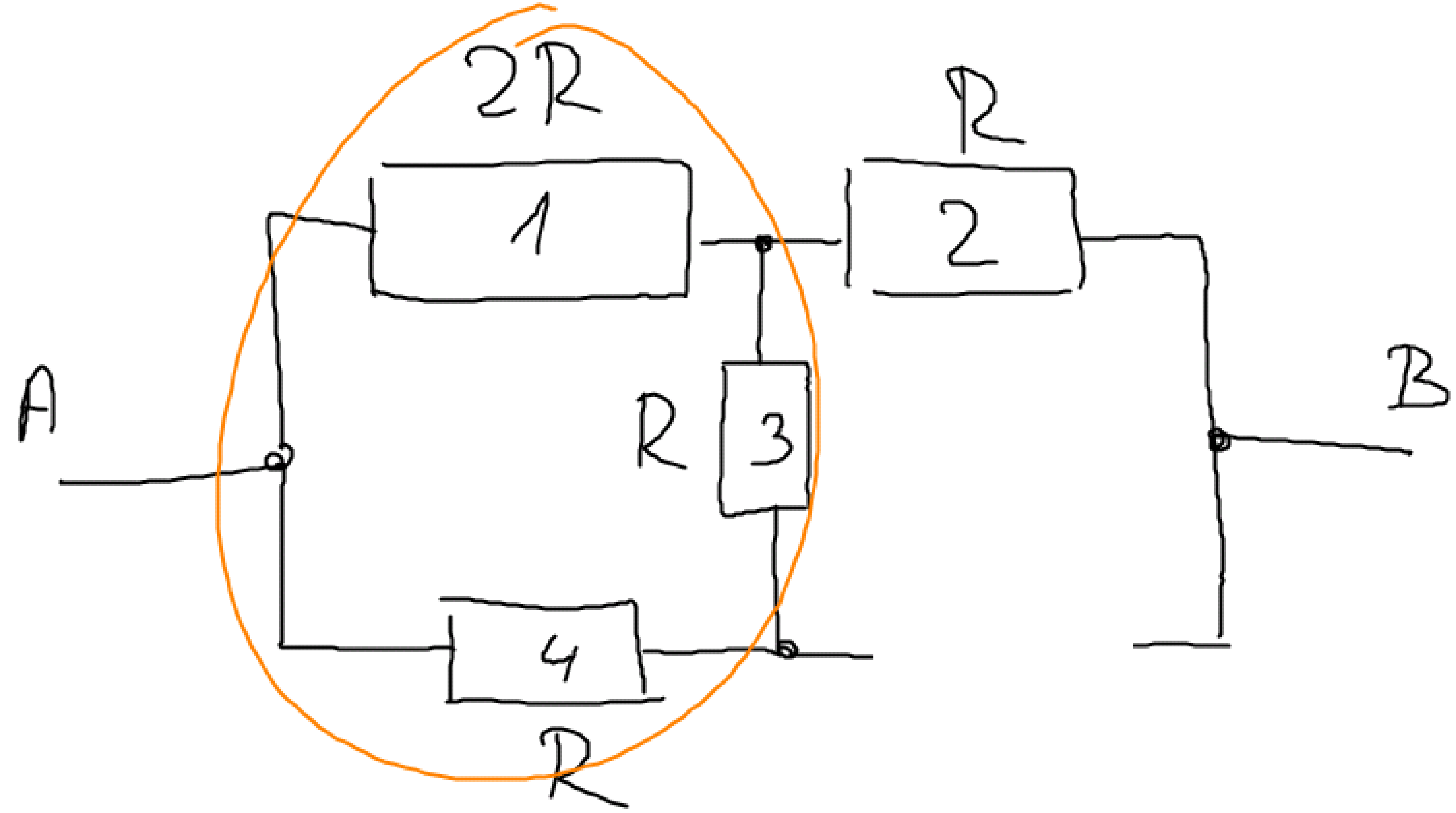
příklad č 4



$$R_{p4} = \frac{2R \cdot R}{2R + R} = \frac{2}{3} R$$

$$R_4 = 2R + \frac{2}{3} R = \frac{8}{3} R$$

připojení se $\underline{5}$:



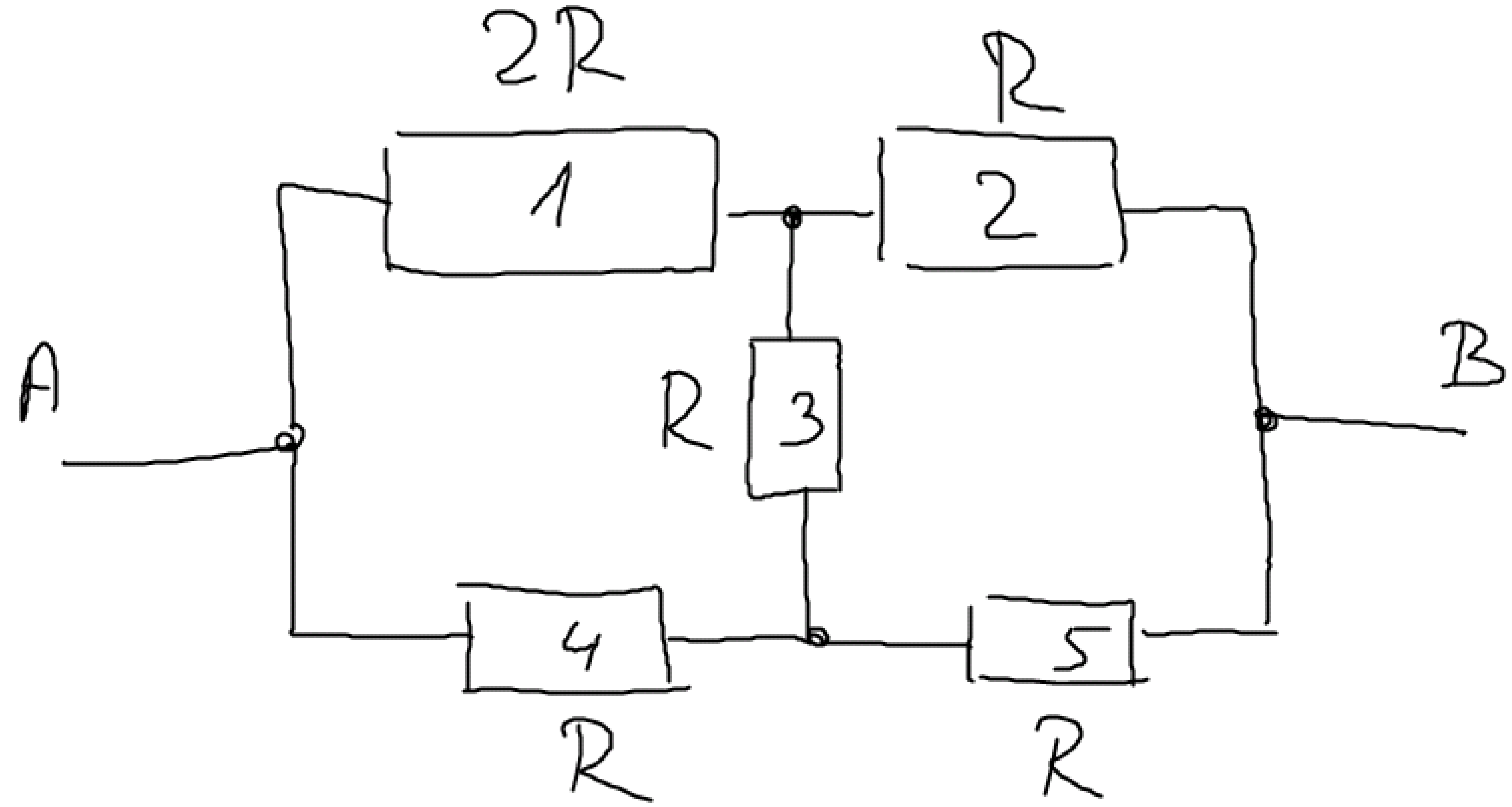
$$R_{p5} = \frac{2R \cdot 2R}{2R + 2R} = R$$

$$R_5 = R + R = 2R$$

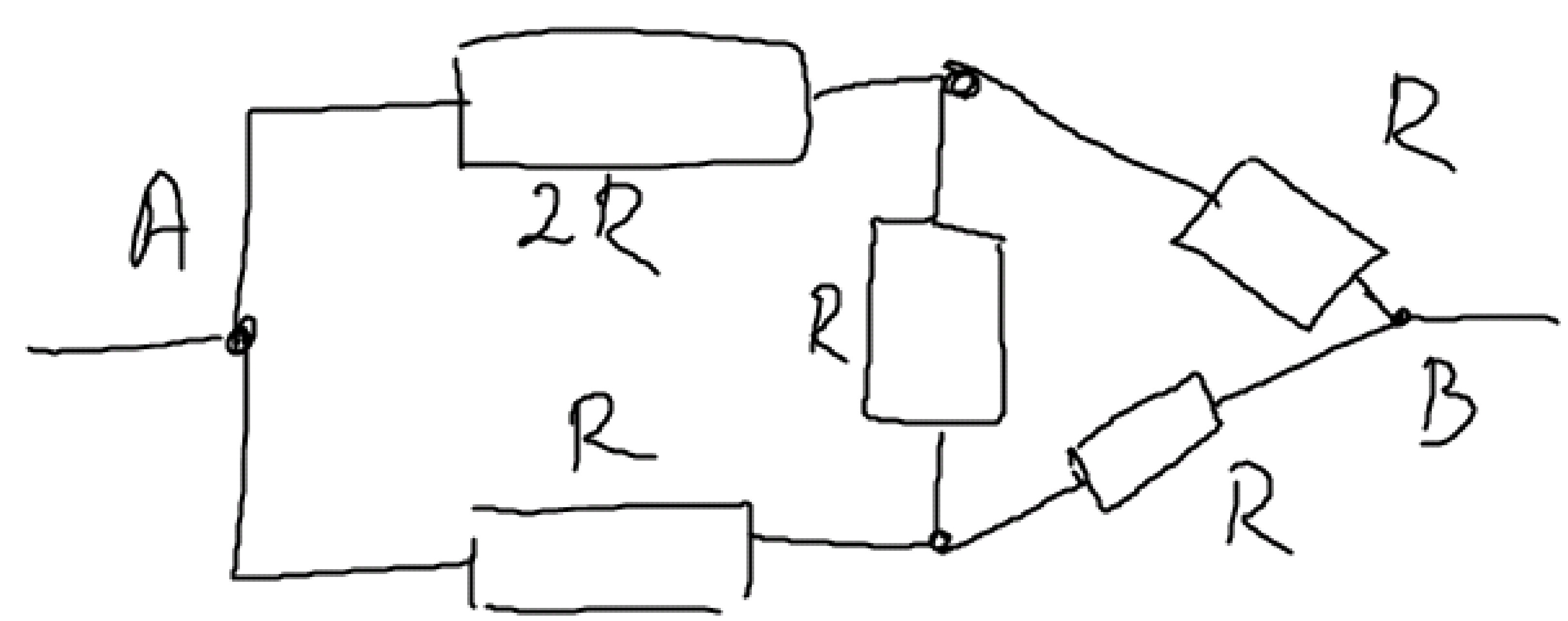
b) celkový odpor je:

- nejmenší $\Leftrightarrow$ poškozen je $\underline{\underline{3}}$
- největší $\Leftrightarrow$ poškozen je $\underline{\underline{4}}$

c)



TROJÚHELNÍK

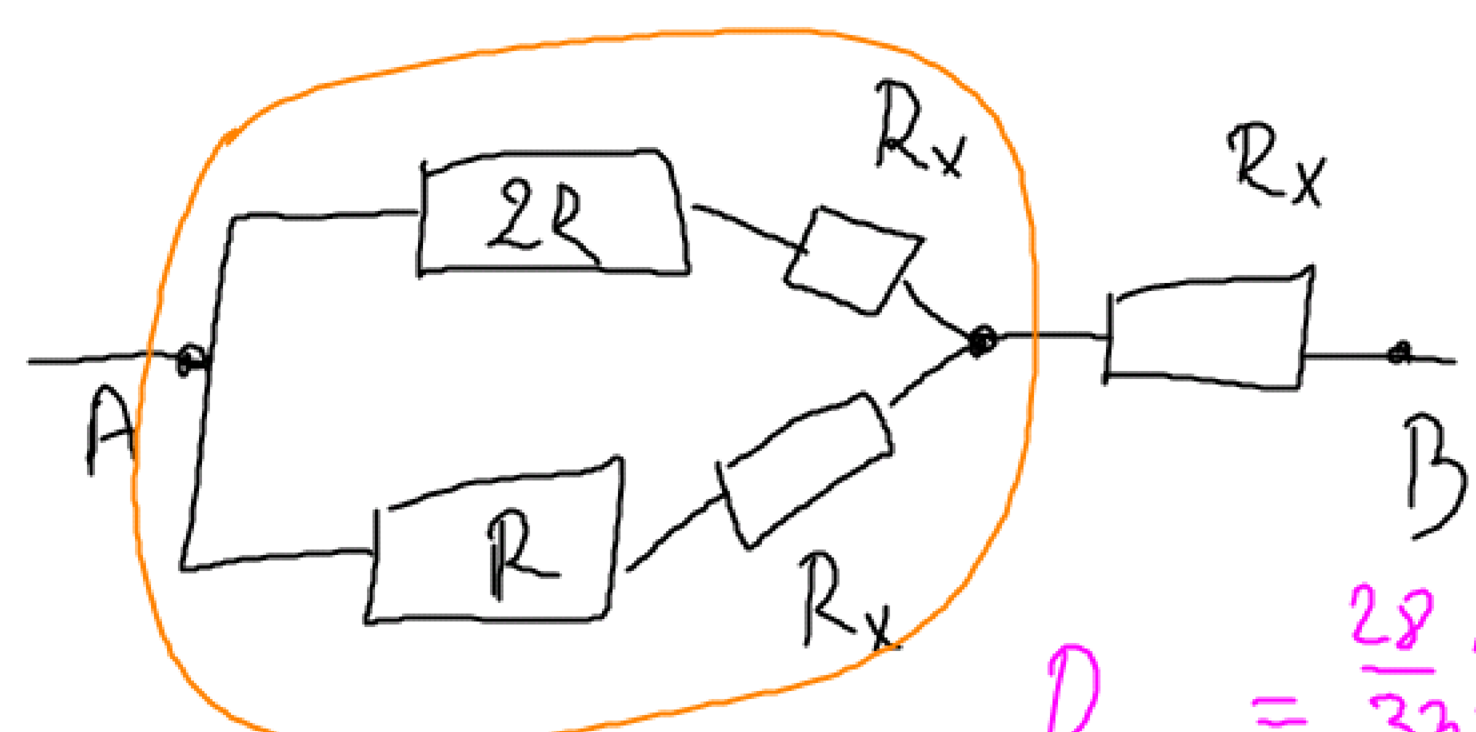


TRANSFIGURACE

$$R_x = \frac{R \cdot R}{R+R+R}$$

$$R_x = \frac{R}{3}$$

HVĚZDA



$$R_p = \frac{\frac{7}{3}R \cdot \frac{4}{3}R}{\frac{7}{3}R + \frac{4}{3}R} = \frac{28}{33}R$$

$$\underline{R_{AB}} = \frac{28}{33}R + \frac{R}{3} = \underline{\frac{13}{11}R}$$

c) nejmenší změna:

$$\Delta R = \frac{6}{5}R - \frac{13}{11}R = \frac{66-65}{55}R = \frac{1}{55}R$$

$$\underline{\underline{\delta R}} = \frac{\Delta R \cdot 100\%}{R_{AB}} = \frac{\frac{1}{55}R}{\frac{13}{11}R} \cdot 100\% = \frac{100}{65}\% = \underline{\underline{1{,}5\%}}$$

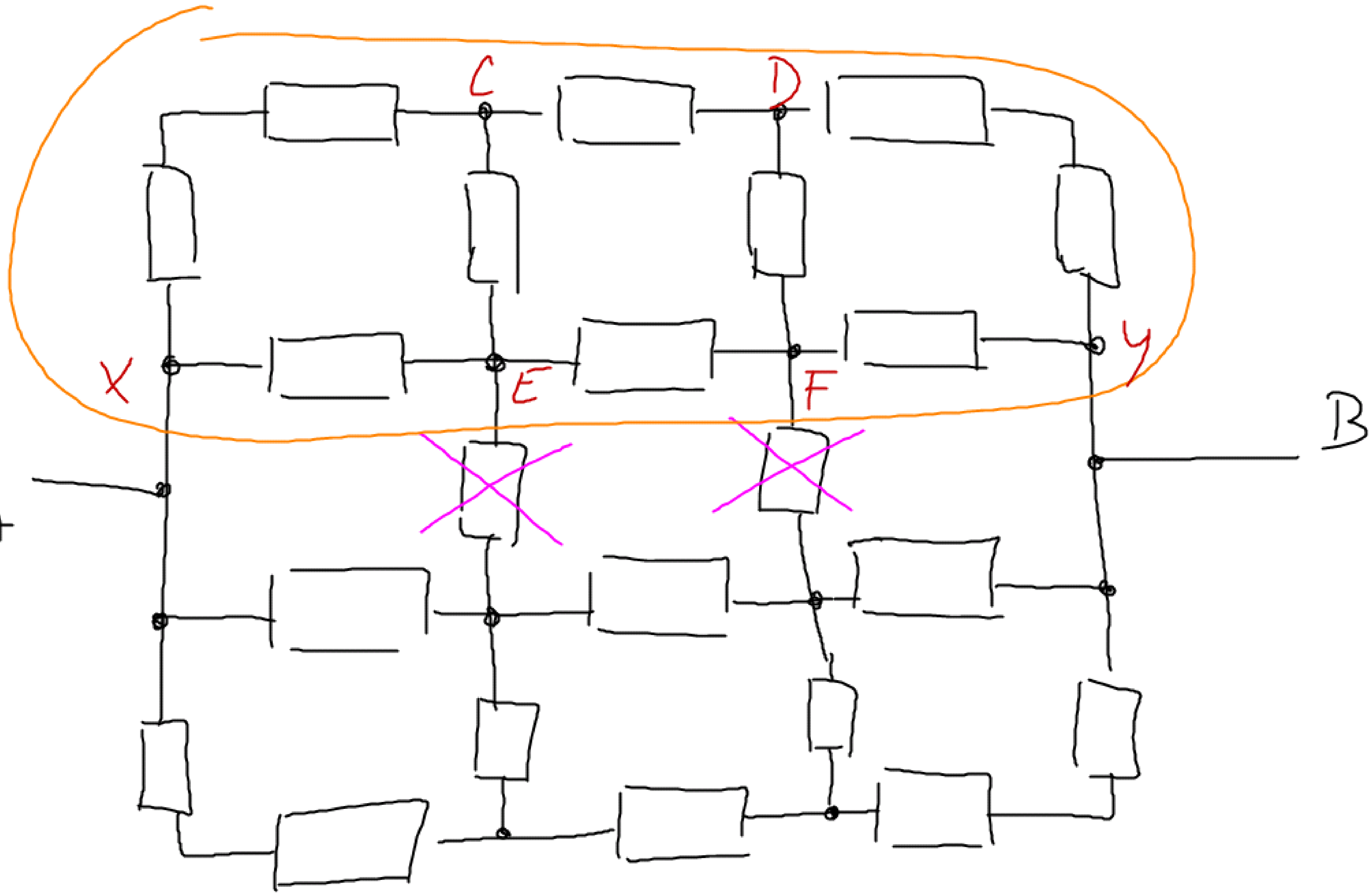
největší změna:

$$\Delta R_2 = \left(\frac{8}{3}R - \frac{13}{11}R\right) = \frac{49}{33}R$$

$$\underline{\underline{\delta R_2}} = \frac{\Delta R_2}{R_{AB}} \cdot 100\% = \frac{\frac{49}{33}R}{\frac{13}{11}R} 100\% = \frac{49 \cdot \cancel{11}}{13 \cdot \cancel{33}_{3}} \cdot 100\%$$

$$= \frac{49}{39} \cdot 100\% = \underline{\underline{126\%}}$$

FO - 57 - 11 - B



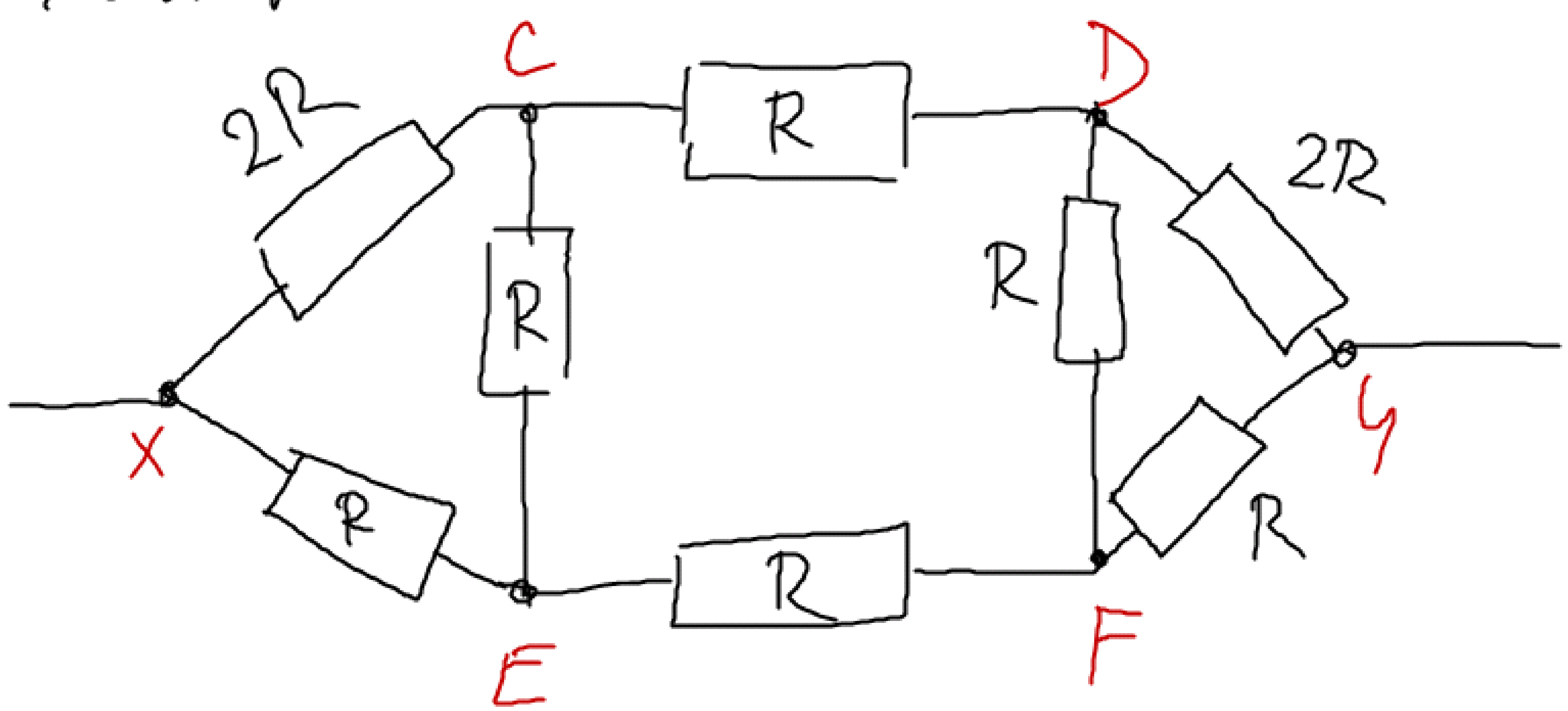
$R_{AB} = ?$

× stejné odpory VŠECH ∧ symetrické zapojení ∧ 1. Kirchhoffův zákon

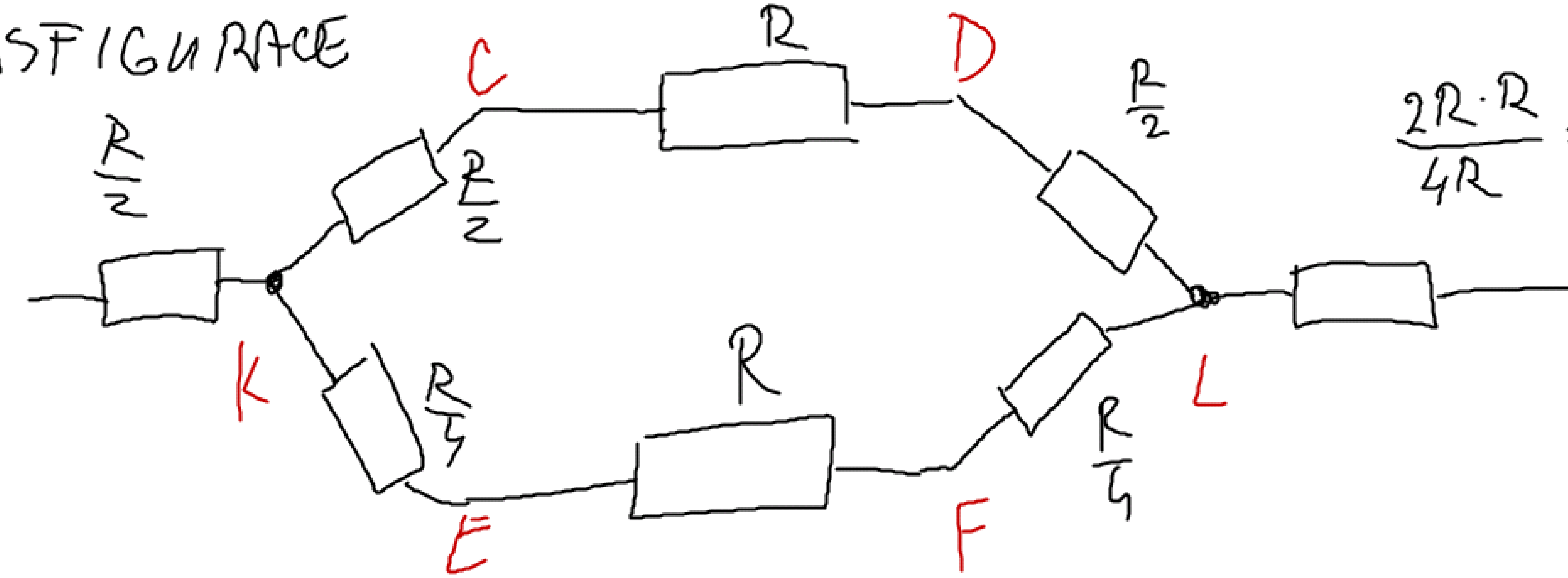
Zapojení je symetrické => stačí vyjádřit

a pak R_{AB} je paralelní zapojení stejných rezistorů

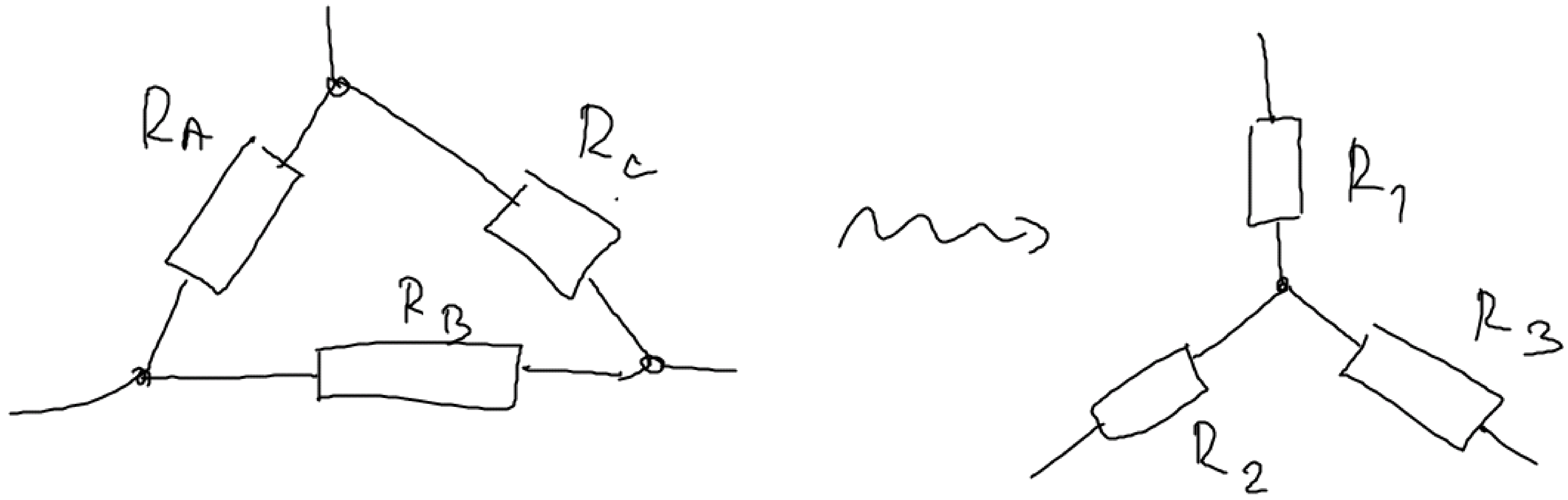
jiná geometrie vodičů, ne zapojení



TRASFIGURACE



$$\frac{2R \cdot R}{4R} = \frac{R}{2}$$



$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_B}{R_A + R_B + R_C}$$

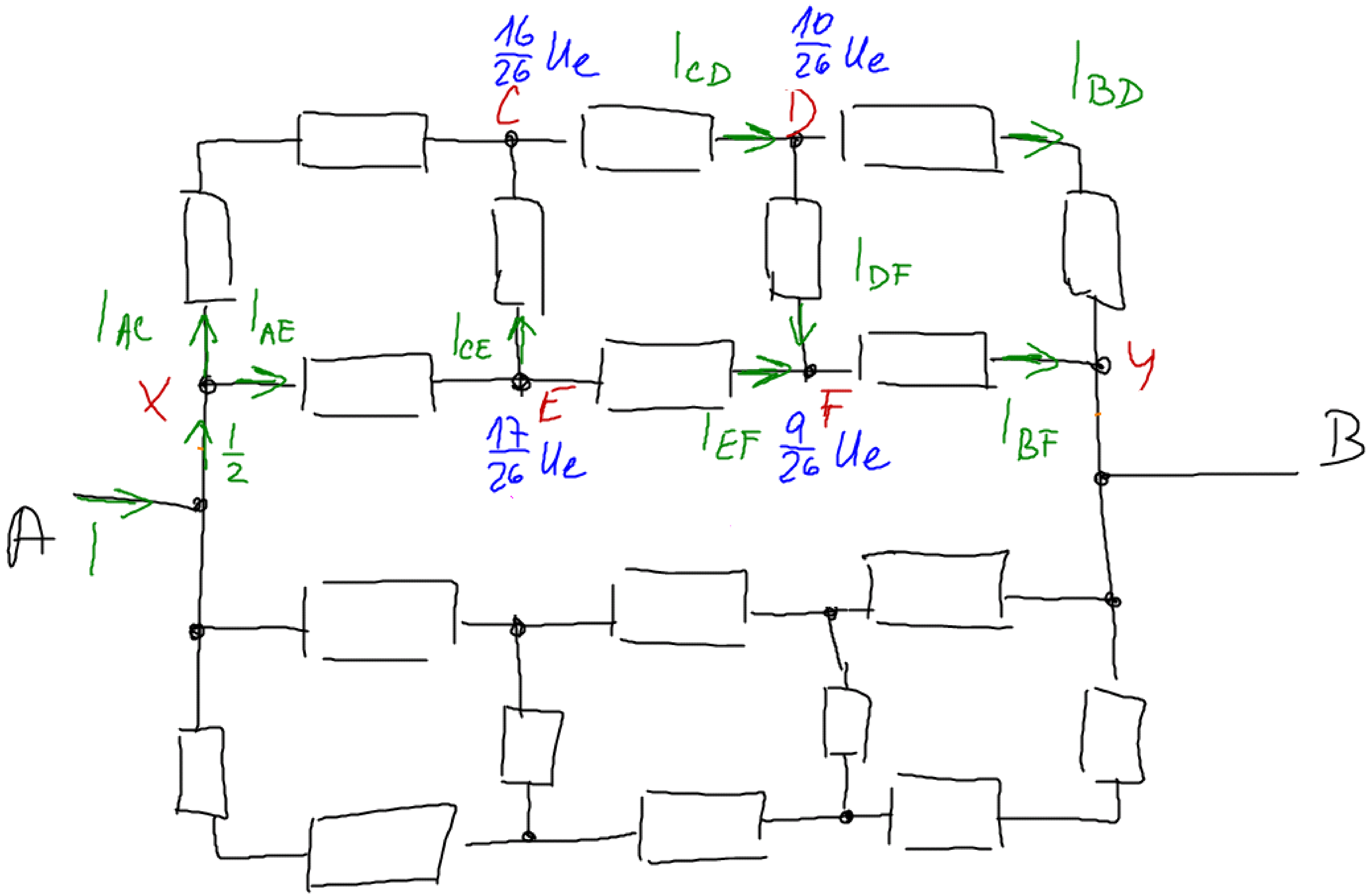
$$R_3 = \frac{R_B R_C}{R_A + R_B + R_C}$$

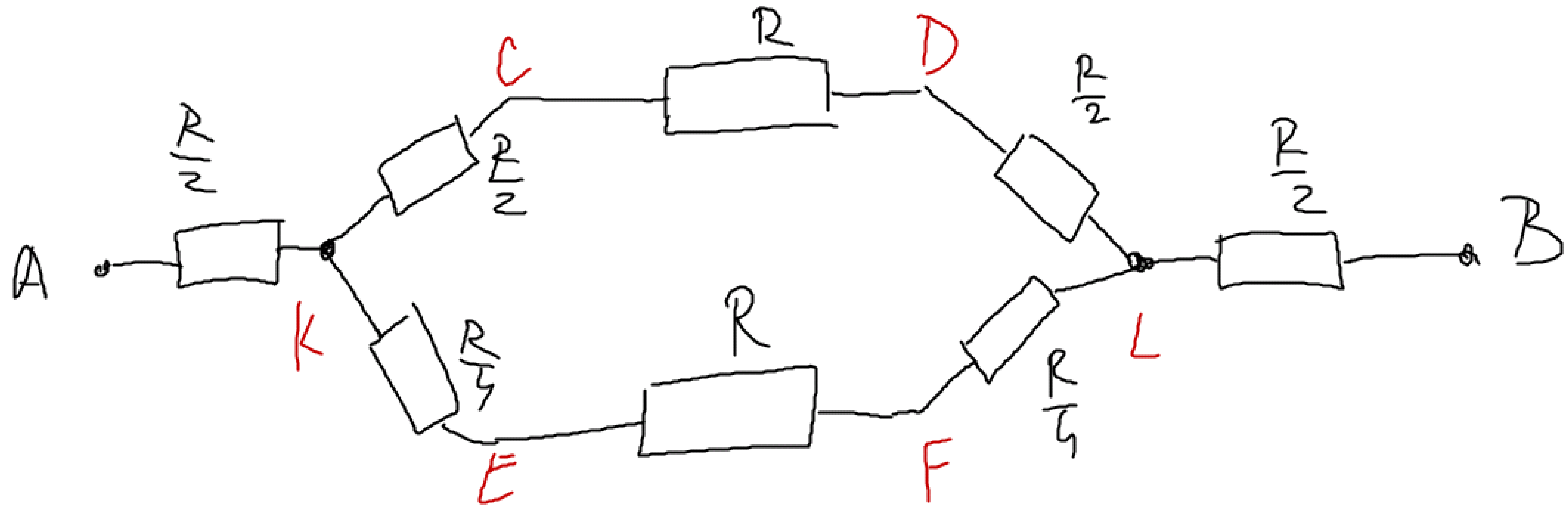
$$R_1 = \frac{R}{2} + \frac{2R \cdot \frac{3}{2}R}{2R + \frac{3}{2}R} + \frac{R}{2} =$$

$$= R + \frac{3R}{\frac{7}{2}} = \frac{13}{7}R$$

$$\underline{\underline{R_{AB}}} = \frac{R_1 \cdot R_1}{R_1 + R_1} = \frac{R_1}{2} = \underline{\underline{\frac{13}{14}R}}$$

k A a B --- zdroj napětí U_e





pomocí potenciálu: volba $\varphi_B = 0$ (BÚNO)

$I_i = ?$

$\varphi_A = U_e$

$$U_{KL} = \varphi_K - \varphi_L = \frac{12}{26} U_e$$

$$\varphi_L = \frac{\frac{R}{2}}{R_1} \varphi_A = \frac{\frac{R}{2}}{\frac{13}{7} R} U_e = \frac{7}{26} U_e$$

$$\varphi_K = U_e - \varphi_L = U_e - \frac{7}{26} U_e = \frac{19}{26} U_e$$

$$\varphi_D = \varphi_L + \frac{U_{KL}}{\frac{R}{2}+R+\frac{R}{2}} \cdot \frac{R}{2} = \frac{7}{26}U_e + \frac{\frac{12}{26}U_e}{2R} \cdot \frac{R}{2} = \frac{10}{26}U_e$$

$$\varphi_C = \varphi_L + \frac{U_{KL}}{\frac{R}{2}+R+\frac{R}{2}} \cdot \frac{3R}{2} = \frac{7}{26}U_e + \frac{\frac{12}{26}U_e}{2R} \cdot \frac{3}{2}R = \frac{16}{26}U_e$$

$$\varphi_F = \varphi_L + \frac{U_{KL}}{\frac{R}{4}+R+\frac{R}{4}} \cdot \frac{R}{4} = \frac{7}{26}U_e + \frac{\frac{12}{26}U_e}{\frac{3}{2}R} \cdot \frac{R}{4} = \frac{9}{26}U_e$$

$$\varphi_E = \varphi_L + \frac{U_{KL}}{\frac{R}{4}+R+\frac{R}{4}} \cdot \frac{5}{4}R = \frac{7}{26}U_e + \frac{\frac{12}{26}U_e}{\frac{3}{2}R} \cdot \frac{5R}{4} = \frac{17}{26}U_e$$

$$I_{AC} = \frac{\varphi_A - \varphi_C}{2R} = \frac{U_e - \frac{16}{26}U_e}{2R} = \frac{5}{26}\frac{U_e}{R}$$

$$I_{CD} = \frac{\varphi_C - \varphi_D}{R} = \frac{6}{26}\frac{U_e}{R} = \frac{3}{13}\frac{U_e}{R}$$

$$I_{BD} = \frac{\varphi_D - \varphi_B}{2R} = \frac{10}{26}\cdot\frac{1}{2}\frac{U_e}{R} = \frac{5}{26}\frac{U_e}{R}$$

$$I_{AE} = \frac{\varphi_A - \varphi_E}{R} = \frac{U_e - \frac{17}{26}U_e}{R} = \frac{9}{26}\frac{U_e}{R}$$

$$I_{EF} = \frac{\varphi_E - \varphi_F}{R} = \frac{8}{26}\frac{U_e}{R} = \frac{4}{13}\frac{U_e}{R}$$

$$I_{BF} = \frac{\varphi_F - \varphi_B}{R} = \frac{9}{26}\frac{U_e}{R}$$

$$I_{CE} = \frac{\varphi_E - \varphi_C}{R} = \frac{1}{26}\frac{U_e}{R}$$

$$I_{DF} = \frac{\varphi_D - \varphi_F}{R} = \frac{1}{26}\frac{U_e}{R}$$

$$I = 2\left(I_{AC} + I_{AE}\right) = 2\cdot\frac{14}{26}\frac{U_e}{R} = \frac{14}{13}\frac{U_e}{R}$$

FO – 57 – 1 – A

$\alpha = 5^\circ$

$n_1 = 1{,}5$

$n_2 = 1{,}7$

$f = 100\ \text{cm}$

---

a) $y = ?$

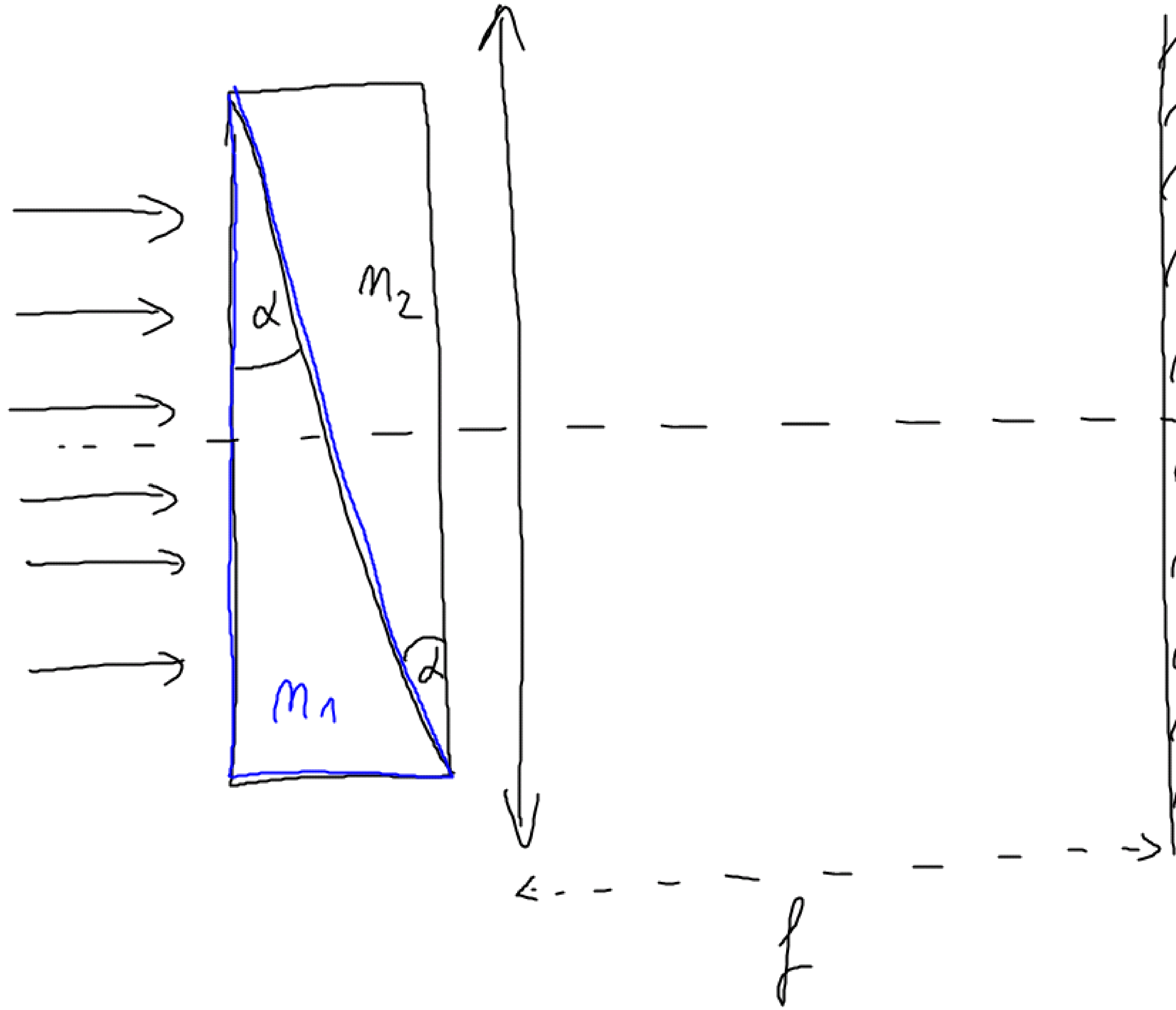
b) $y_1 = ?$

c) $y_2 = ?$

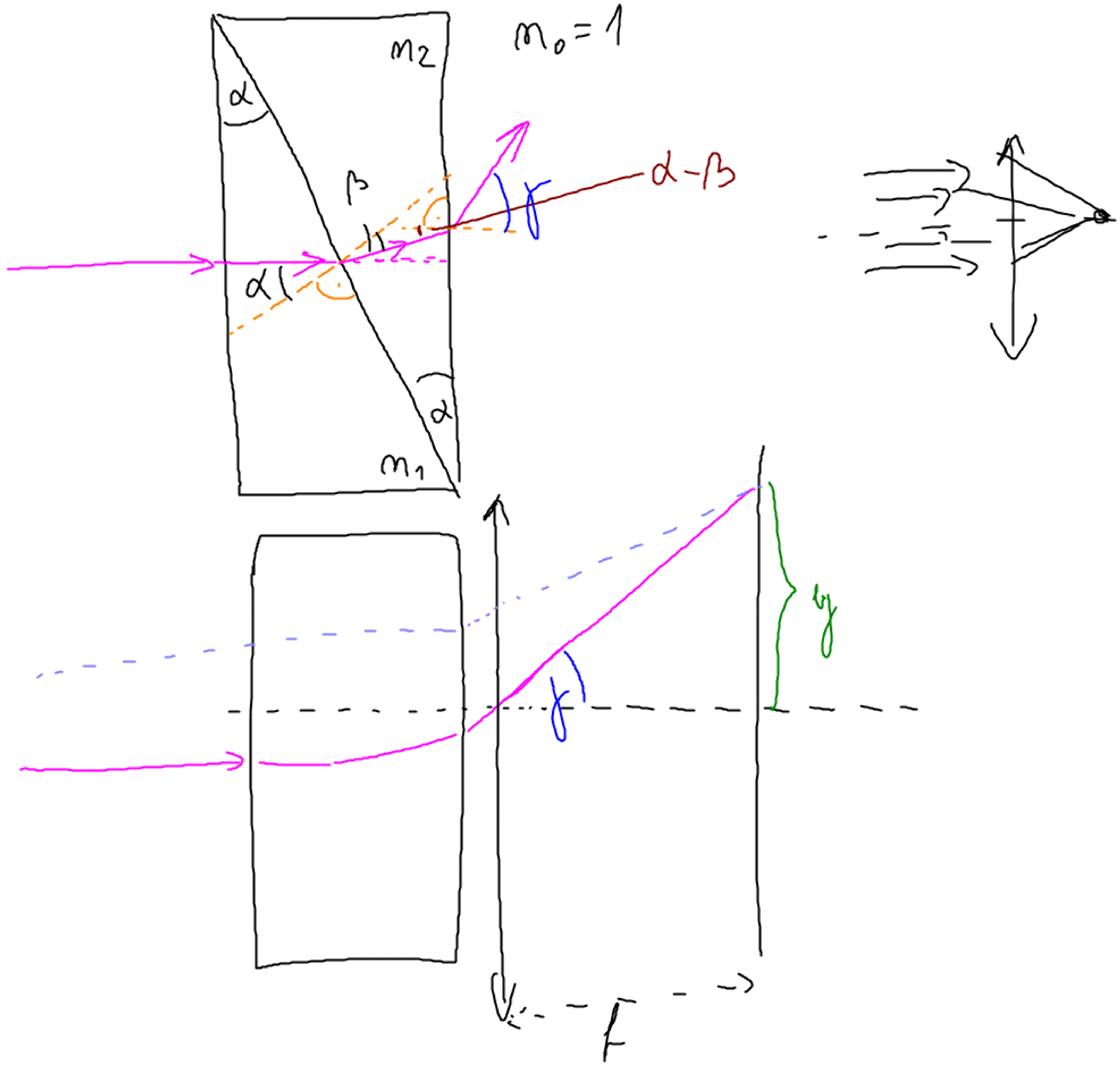
$\sin x \doteq x$

$\Downarrow$

$\operatorname{tg} x \doteq x$



a)



$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1} \qquad \frac{\sin(\alpha-\beta)}{\sin\gamma} = \frac{n_0}{n_2}$$

$$\sin\beta = \frac{n_1}{n_2}\sin\alpha \qquad \sin\gamma = \frac{n_2}{n_0}\sin(\alpha-\beta)$$

$$\beta = 4{,}41^\circ \qquad \gamma = 1{,}002^\circ$$

$$\operatorname{tg}\gamma = \frac{y}{f} \Rightarrow \cancel{y} = f\operatorname{tg}\gamma = \underline{\underline{1{,}75\ \text{cm}}}$$

ZANEDBÁNÍ:

$$\frac{\alpha}{\beta} = \frac{n_2}{n_1}$$

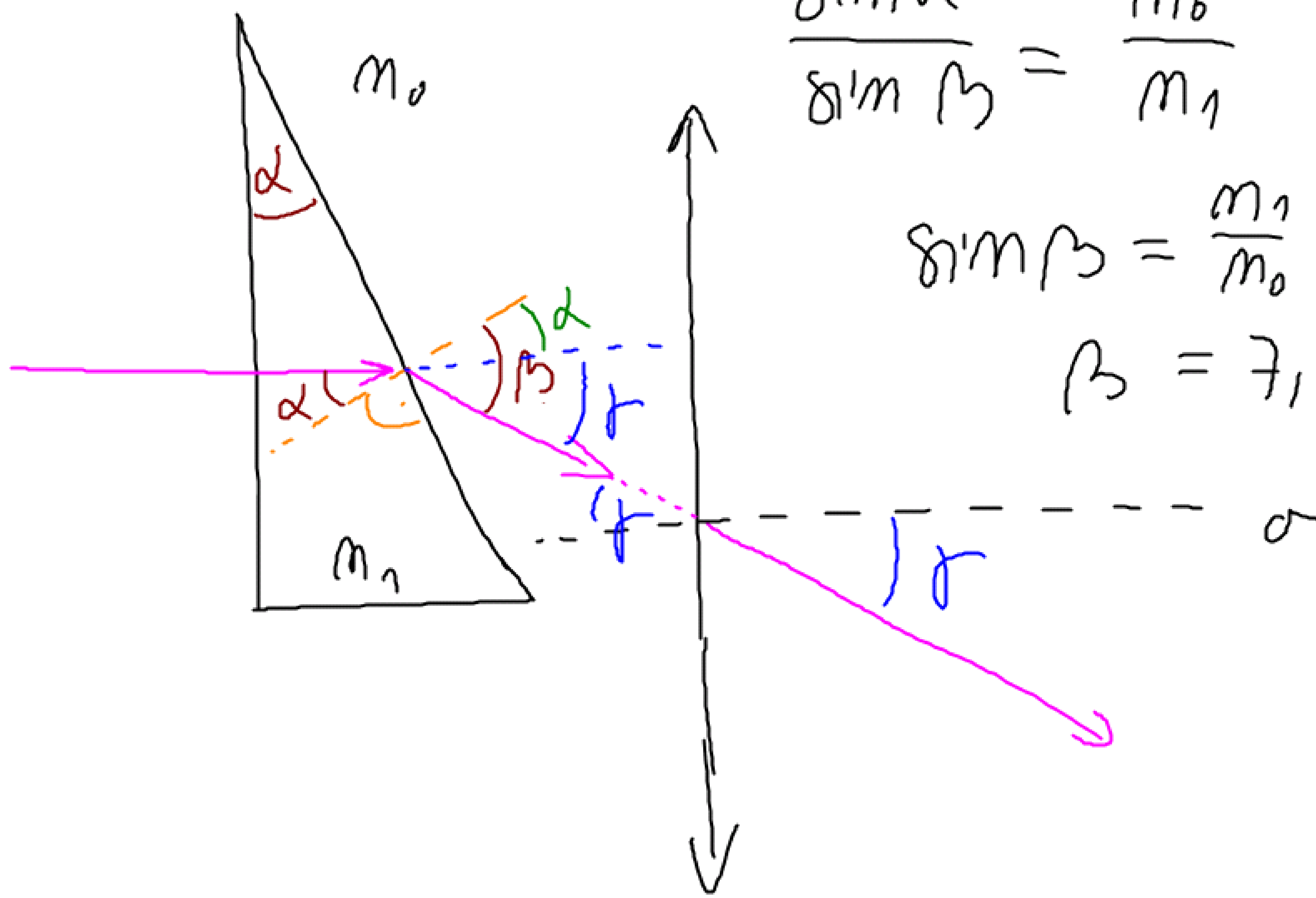
$$\beta = \frac{n_1}{n_2}\alpha \quad \text{RADIÁNY}$$

$$\frac{\alpha-\beta}{\gamma} = \frac{n_0}{n_2}$$

$$\gamma = \left(\alpha - \frac{n_1}{n_2}\alpha\right)\frac{n_2}{n_0} = \frac{n_2-n_1}{n_0}\alpha$$

$$\cancel{y} = f\gamma = f\,\frac{n_2-n_1}{n_0}\alpha = \underline{\underline{1{,}74\ \text{cm}}}$$

b)



$$\frac{\sin\alpha}{\sin\beta} = \frac{n_0}{n_1}$$

$$\sin\beta = \frac{n_1}{n_0}\sin\alpha$$

$$\beta = 7{,}5^\circ$$

$$\gamma = \beta - \alpha$$

$$\gamma = 2{,}5^\circ$$

$$\operatorname{tg}\gamma = \frac{y_1}{f}$$

$$y_1 = f\operatorname{tg}\gamma$$

$$\underline{\underline{y_1 = 4{,}39\,\mathrm{cm}}}$$

ZANEDBÁNÍ:

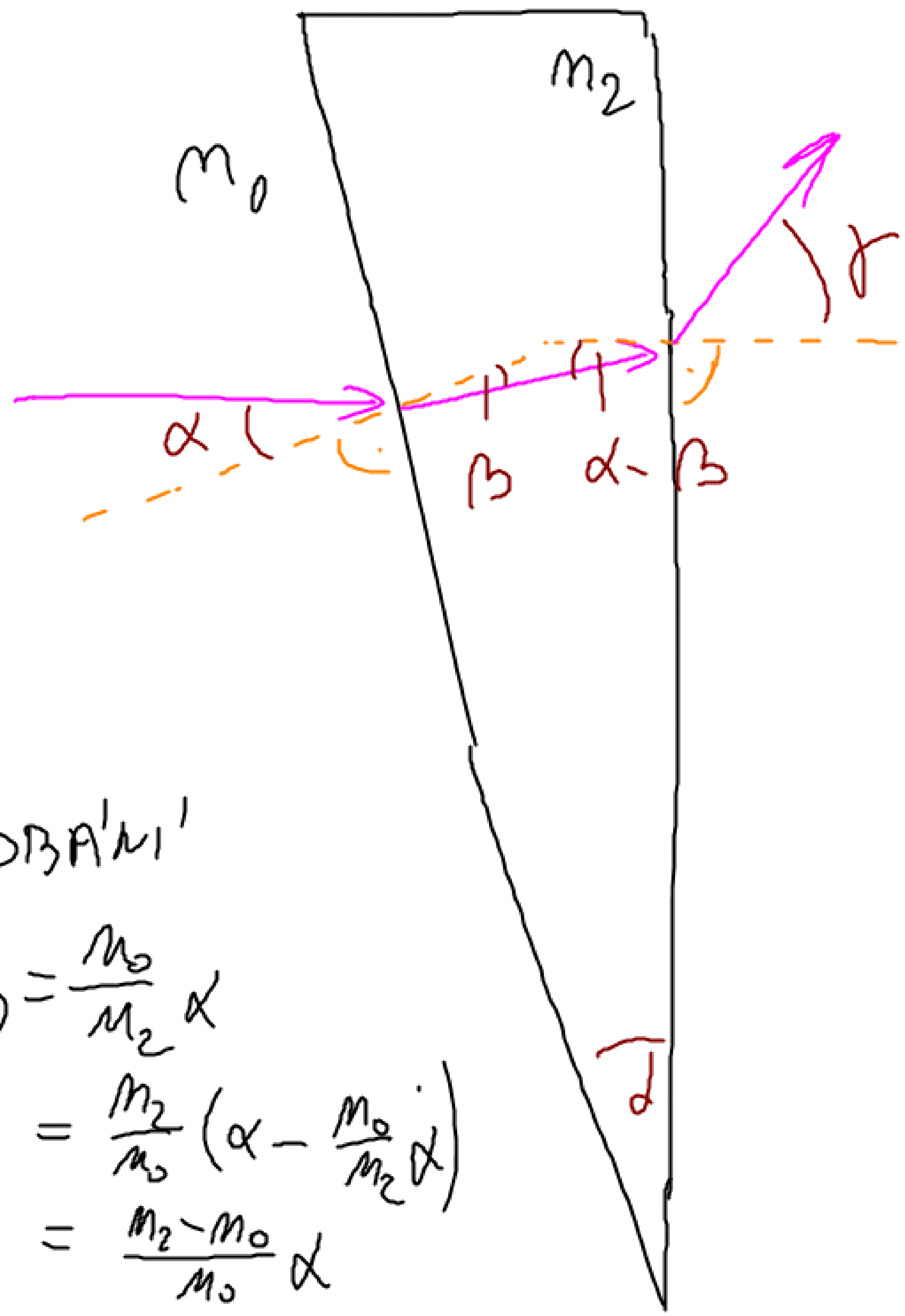
$$\frac{\alpha}{\beta} = \frac{n_0}{n_1}$$

$$\beta = \frac{n_1}{n_0}\alpha$$

$$\gamma = \alpha\left(\frac{n_1}{n_0} - 1\right)$$

$$\underline{y_1} = f\cdot\alpha\left(\frac{n_1}{n_0} - 1\right) = \underline{\underline{4{,}36\,\mathrm{cm}}}$$

c)



$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_0}$$

$$\sin\beta = \frac{n_0}{n_2}\sin\alpha$$

$$\beta = 2{,}54^\circ$$

$$\frac{\sin(\alpha-\beta)}{\sin\gamma} = \frac{n_0}{n_2}$$

$$\sin\gamma = \frac{n_2}{n_0}\sin(\alpha-\beta)$$

$$\gamma = 3{,}5^\circ$$

$$\operatorname{tg}\gamma = \frac{y_2}{f} \Rightarrow y_2 = f\operatorname{tg}\gamma$$

$$\underline{\underline{y_2 = 6{,}13\ \text{cm}}}$$

ZANEDBÁNÍ

$$\beta = \frac{n_0}{n_2}\alpha$$

$$\gamma = \frac{n_2}{n_0}\left(\alpha - \frac{n_0}{n_2}\alpha\right)$$

$$\gamma = \frac{n_2-n_0}{n_0}\alpha$$

$$\underline{y_2} = f\cdot\frac{n_2-n_0}{n_0}\alpha = \underline{6{,}11\ \text{cm}}$$

FO – 53 – 1 – A

$^{235}_{92}U$ ... $p_1 = 0{,}72\%$ .... $T_1 = 7{,}038 \cdot 10^8$ let

$^{238}_{92}U$ $T_2 = 4{,}468 \cdot 10^9$ let

a) $m = 1$ kg

$A = ?$

$$N = \frac{m}{M_m} \cdot N_A \cdot \frac{p}{100}$$

$$A = \lambda N = \frac{\ln 2}{T} N$$

$$\underline{\underline{A = 1{,}29 \cdot 10^7 \text{ Bq}}}$$

$$A = A_1 + A_2 = \ln 2 \cdot \left(\frac{N_1}{T_1} + \frac{N_2}{T_2} \right) =$$

$$= \frac{m \cdot N_A \cdot \ln 2}{100} \left(\frac{p_1}{M_{m_1} T_1} + \frac{100 - p_1}{T_2 \cdot M_{m2}} \right) =$$

$$= \frac{1}{365{,}25 \cdot 24 \cdot 3600} \cdot \frac{1 \cdot 6{,}022 \cdot 10^{23} \ln 2}{100} \left(\frac{0{,}72}{235 \cdot 10^{-3} \cdot 7{,}038 \cdot 10^8} + \frac{99{,}28}{238 \cdot 10^{-3} \cdot 4{,}468 \cdot 10^9} \right) \text{Bq}$$

b) $t = 1{,}9 \cdot 10^9$ let

$$N = N_0 \cdot 2^{-\frac{t}{T}} = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

$$N_{01} = N_1 \cdot 2^{\frac{t}{T_1}}$$

$$N_{02} = N_2 \cdot 2^{\frac{t}{T_2}}$$

$$\frac{N_{01}}{N_{02}} = \frac{N_1}{N_2} \cdot 2^{t\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

$$N = \frac{m}{M_m} \cdot N_A$$

$$m = \frac{N \cdot M_m}{N_A}$$

$$\frac{m_1}{m_2} = \frac{N_{01} M_{m1}}{N_{02} M_{m2}} = \frac{p_1}{100 - p_1} \cdot 2^{t\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} = \frac{1}{28{,}5}$$

$$m_1 + m_2 = m$$

$$p_{01} = \frac{m_1}{m}$$

$$28{,}5 m_1 + m_1 = m$$

$$m_1 = \frac{m}{29{,}5}$$

$$p_{01} = \frac{1}{29{,}5} \qquad p_{02} = 1 - \frac{1}{29{,}5}$$

c) $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{143}_{56}Ba + ^{90}_{36}Kr + 3\,^{1}_{0}n$

$$E_V = \Delta m \cdot c^2 = \left(m_U + m_n - m_{Ba} - m_{Kr} - 3\,m_n\right)c^2$$

$$= 2{,}785 \cdot 10^{-11}\,J = 174\ MeV$$

d) $E_1 = 200\ MeV$

$E_{AC} = 0{,}36\ GJ$

$m_r = 5\ t$

$$N = \frac{m_r}{M_m} N_A = \frac{5 \cdot 10^3}{235 \cdot 10^{-3}} \cdot 6{,}022 \cdot 10^{23} = 1{,}28 \cdot 10^{28}$$

$$E = N \cdot E_1 = 1{,}28 \cdot 10^{28} \cdot 200 \cdot 10^6 \cdot 1{,}602 \cdot 10^{-19}\ J = 4{,}1 \cdot 10^{17}\ J$$

$$E = 4{,}1 \cdot 10^8\ GJ$$

$\Rightarrow 10^9$ lidí